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To Boloni.

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Summary

This dissertation is concerned with theory and behavior in stopping problems.

In a stopping problem an agent or individual observes the realization of some exogenous and stationary stochastic process over time. At every point in time, she has the right or the once-only option to stop the process in order to earn a function of the past and current values of the process. The agent's objective then is to choose the point in time to exercise the option in order to maximize an expected reward or to minimize an expected loss. Such problems constitute the most rudimentary, yet truly dynamic class of choice problems that is studied in economics.

More formally, let time be discrete $t = 1, 2, \dots$ and denote by $X = (X_1, X_2, \dots)$ a sequence of random variables which we will refer to as the payoff or offer process the agent faces. In the first two chapters, we will assume that the probability law which generates realizations of these random variables is known. The third chapter will weaken this assumption. Associated with a given sequence of realizations $\hat{X} = (X_1 = x_1, X_2 = x_2, \dots)$ of X , which we will sometimes refer to as a *path* of X , is a sequence of real-valued earnings or consequences from stopping $V(\hat{X}) = (v_1(X_1 = x_1), v_2(X_1 = x_1, X_2 = x_2), \dots)$. After observing the first t realizations $(X_1 = x_1, X_2 = x_2, \dots, X_t = x_t)$, the agent may decide to stop and earn $v(X_1 = x_1, X_2 = x_2, \dots, X_t = x_t)$ or to continue and observe X_{t+1} and face the same decision again. Here v_t may be any real-valued function of X . For example, suppose the payoff from stopping in period t is the current value $X_t = x_t$, then v_t could be a standard Bernoulli utility $v_t(X_1 = x_1, \dots, X_t = x_t) = u(x_t)$. Similarly, suppose the payoff from stopping in period t is the sum of all future payoffs, e.g. when investing into a widget that generates profits once installed, then v_t may be a von Neumann-Morgenstern expected utility functional $\mathbb{E}[\sum_{s=t}^T u(X_s) \mid \hat{X}]$.

Under mild regularity conditions (see e.g. Chow, Robbins & Siegmund, 1971), an optimal stopping rule exists and may be found based on Bellman's Principle of Optimality. That is, instead of finding the optimal rule for the multi-period problem,

i.e. the whole path of X , the optimization problem is reduced to the subproblem of choosing between stopping and continuation at each point in time. Hence, the optimal rule may be found by choosing for every t between the immediate gains from stopping $v(X_1 = x_1, X_2 = x_2, \dots, X_t = x_t)$, and the expected future gains given that the agent behaves optimally for all future X_t . For finite-horizon problems, this amounts to applying backward induction, whereas in stopping problems with unknown or infinite horizon, recursive dynamic programming can be used to derive the optimal rule.

Historically, stopping problems of this form first occurred in statistical decision theory (see Wald, 1945a; Arrow, Blackwell & Girshick, 1948) and were further laid out in the subsequent books by Abraham Wald: *Sequential Analysis* in 1947 and *Statistical Decision Functions* in 1950.

A key example for an application of stopping problems to economics is job search. Job seekers can be thought to face a stopping problem, as they sequentially receive wage offers over time and have the option to accept the current offer or to reject it in favor of future offers (Stigler, 1962; McCall, 1970; Rogerson, Shimer & Wright, 2005). Based on the work by e.g. Diamond (1982), stopping theory has formed a crucial building block in the development of modern labor market models (Mortensen & Pissarides, 1994, 1999). Further applications arise in many other fields. In finance holders of an American call or put options can be thought to face a stopping problem, because they have the right to exercise, i.e. buy or sell, the underlying asset at a fixed price any time before some maturity date (Jacka, 1991). Similarly, firms that face a market-entry decision or an irreversible investment or liquidation decision also can be thought to face a stopping problem (McDonald & Siegel, 1986a; Dixit & Pindyck, 1994a). Stopping problems, however, also occur in more mundane or every-day situations. Examples are the buying or selling of a house or renting an apartment, searching for the best price for a new TV or deciding to go for a Ph.D. degree or not.

Given their widespread application to economic theory and their frequent occurrence in every-day life, stopping problems are of considerable interest. While the theoretical literature on optimal stopping is vast, little is known about whether the theoretical models of behavior describe actual behavior in stopping problems well or at least provide a pertinent approximation. Moreover, even though the theoretical literature on optimal stopping problems is very advanced, it is almost exclusively concerned with the case where the objective function of the expected

utility (EU) type. While EU is the leading normative theory of behavior in economics, it is frequently found to be descriptively deficient.

In the first chapter of this dissertation, we therefore revisit the theoretical backdrop and provide new theoretical results about optimal stopping both under EU and two prominent behavioral theories: Kahneman & Tversky's prospect theory preferences without probability weighting and Loomes & Sugden's regret preferences. Under EU it is a well-known result that the optimal rule is simple: Stop as soon as the payoff process attains a certain reservation level. Otherwise wait. Our first contribution is to show that this indeed holds under a very general notion of EU. The way we generalize this result relative to the literature, is by relaxing a standard monotonicity assumption to a single-crossing assumption on the Bernoulli utility function of an agent. While our single-crossing assumption holds for any strictly increasing and concave utility, the monotonicity assumption does, e.g., not necessarily hold under constant absolute risk aversion. However, the gain from relaxing the classical assumption on utility is more substantial.

First, utility need not be concave, but can be neither concave nor differentiable under our assumption. This allows us to extend our results to value functions or gain-loss utility (Kahneman & Tversky, 1979a).¹ Our work relates to several recent papers recent theoretical papers that are concerned with optimal stopping rules under behavioral preferences. For example, Barberis & Xiong (2009, 2012); Henderson (2012); Ebert & Strack (2012); Xu & Zhou (2013) apply or derive optimal stopping rules for an agent with gain-loss preferences or cumulative prospect theory (CPT) preferences – the leading positive theory of choice under uncertainty. However, in dynamic settings the application of prospect theory and gain-loss utility remains a delicate issue and especially in combination with probability weighting is found to easily run into conceptual and theoretical problems or not to yield behaviorally different predictions than EU (see e.g. Hens & Vlcek, 2011; Ebert & Strack, 2012). And in fact, we show that in the classical setting we consider, gain-loss preferences do not yield behaviorally different predictions than EU.

Second, under the single-crossing assumption we show that cut-off strategies

¹We will use the term value function and gain-loss utility synonymously to describe an agent whose utility is defined over final wealth relative to a fixed and known reference point, e.g. the famous S-shaped utility. To avoid confusion with the term value function found in the dynamic programming literature, we will henceforth use the term gain-loss utility. We do not consider probability distortion of any sort here (see Ebert & Strack, 2012; Xu & Zhou, 2013, for results with probability distortion).

are not only optimal for a larger class of utility functions, but also if one considers off-equilibrium optimal strategies, i.e. subgame perfect strategies. That is, the optimal strategy of an EU agent and an agent with gain-loss utility is robust to erroneous deviations. Even if the agent reaches a point she never planned to reach ex ante, she will not reconsider her subsequent plan.

We show this is different under regret. Regret preferences as we will understand them here were introduced by Loomes & Sugden (1982) in a static one-shot setting.² While much attention has been devoted to the impact of regret on decision making in the psychology (Gilovich, Medvec & Kahneman, 1998; Zeelenberg, van Dijk, Van der Pligt, Manstead, Van Empelen & Reinderman, 1998; Zeelenberg, Van Dijk, Manstead & van der Pligt, 2000; Gilbert, Morewedge, Risen & Wilson, 2004) and neuroscience literature (Camille, Coricelli, Sallet, Pradat-Diehl, Duhamel & Sirigu, 2004; Coricelli, Critchley, Joffily, O'Doherty, Sirigu & Dolan, 2005; Coricelli, Dolan & Sirigu, 2007), the choice theoretic literature that investigates the theoretical prediction under regret – especially in a dynamic context – is still fairly infant.³ To the best of our knowledge the only papers that take regret theory to a dynamic setting are due to Krähmer & Stone (2012) and Hayashi (2009, 2011). While Krähmer & Stone (2012) treat the case of a finite-horizon choice problem under regret à la Loomes & Sugden, Hayashi (2009, 2011) is concerned with commitment and consistency issues in a finite-horizon stopping problem where the agent has minimax-regret preferences in the sense of Wald-Savage. We show that regret preferences à la Loomes & Sugden do not yield behaviorally different predictions in our setting, unless agents can unexpectedly deviate from their ex-ante plan. That is, (i) the ex ante optimal stopping rule for a regret

²Wald (1945b) and Savage (1951) already delineated a model of decision making under regret, often called (*Savage's*) *minimax regret* or *Wald's maximin* model, as a theory of *distribution-free* decision making in statistics. Milnor (1954); Stoye (2011) provide an axiomatization. Under minimax regret the decision maker is completely ignorant of any probabilities that certain states of the world realize and his sole objective is to minimize his maximal regret. This makes minimax regret optimization robust to model misspecification or misperception of the relevant probabilities, by being entirely independent of it or – equivalently – by considering the optimal rule under every possible distribution (see e.g. Hansen & Sargent, 2001, for a related literature in macroeconomics). A very much related robustification comes in the form of the maximin expected utility model due to Gilboa & Schmeidler (1989).

³Nonetheless, there is a growing literature that considers the impact of emotions and counterfactual thinking on choice behavior. For example, see Rabin (2004); Battigalli & Dufwenberg (2007, 2009) or Bordalo, Gennaioli & Shleifer (2012). Examples for applications of minimax-regret preferences to economics are Bergemann & Schlag (2011) who consider monopoly pricing, Filiz-Ozbay & Ozbay (2007) who consider auctions and Linhart & Radner (1989) who consider bargaining under minimax-regret.

is again a cut-off rule and (ii) in a setting where agents always implement their period-0 optimal rule, regret cannot be distinguished from EU. In a setting where agents can unexpectedly deviate from their initial plan, they become ex-post distinguishable from EU. In contrast to EU or gain-loss preferences, a regret agent reconsiders the ex-ante plan in the light of past events and adapts her ex-ante cut-off upward to the value of the past maximum of the payoff process. We call this a disposition to gamble for resurrection.

In the second chapter, we attempt to test some of key implications derived in the first chapter. Based on the setting that was considered there, we designed an experiment which *exactly* replicates the theoretical setup. That is, we face subjects in the laboratory with the option to stop a multiplicative binomial random walk. Given our results from chapter one, we would expect subjects to (i) stop the process at roughly the same reservation level over several repetitions of the same stopping task and (ii) not to stop the process at a point they had seen before. We present the task to subjects in a intuitive graphical representation, much like a stock price on a ticker tape. Subjects observe 65 different random walks and may stop each of the processes at any point in time to obtain a material payoff of $X_t - K$, where $K > 0$ is known. Every process has a random duration, i.e. at some unknown point in time, the process and the option to stop expires. We find almost no evidence for cut-off behavior in our data, i.e. subjects do not seem to play or converge to a unique reservation level and they behave time-inconsistently most of the time. However, beginning with Mosteller & Nogee (1951), individual subjects in experiments are found to exhibit frequent choice switching between alternatives across identical trials. This quite immediately leads one to consider a model of stochastic choice. In principle, most if not all deviations from EU or any other structural model of choice can be explained by a random or stochastic component. That is, while some choice switches may be explained by the structural part representing preferences, the remainder is attributed to random preferences, mistakes or unobservable characteristics. A stochastic choice model as we model it here can be viewed as an econometric model of choice, where an agent chooses the preferred action from a set of available actions not with certainty but with a certain probability. This probability is a function of the utility that arises from choosing the given action relative to all other actions (see Wilcox, 2008, for a primer). In our setting, this means that choice depends on the difference between the value from stopping and continuation at each point. Since our results from

chapter one also hold in a setting where agents deviate from their ex-ante plan, our expressions for the continuation and stopping value remain intact. This allows us to translate our theoretical results into a structural econometric model without any conceptual loss. Additionally, an agent's choice is now affected by a random component that is added to the difference between stopping and continuation value. Different distributional assumptions about this error constitute different choice models. For example, under the assumption that the composite error that hits the utility difference is normally distributed, one speaks of a dynamic probit model, whereas under a logistic distribution one understands the model to be a dynamic version of the standard logit model. Fitting such a model to our data, we not only accommodate for the stochastic component of it, but we may also test the regret model against the EU model. And in fact, our results suggest that a model including regret aversion fits the data substantially better. Intuitively, this is explained by the fact that subjects in our sample seldom stop below the past maximum of the process, i.e. are reluctant to realize a loss relative to past maximum. The structural part of the regret model accommodates for this through the prediction that agents will gamble for resurrection. Intuitively, the regret model yields a better signal-to-noise ratio. It fits the data better, because it can attribute more of the choice switches to its systematic or structural part, as opposed to the unsystematic or stochastic part, than an EU model.

In the third and last chapter, we investigate stopping behavior in a setting, where the probability law that drives the process X_t is not perfectly known to the decision maker. Assuming perfect knowledge of the probabilities of future events seems unrealistic in most (if not all) real-world contexts. Beginning with Becker & Brownson (1964), the impact of ambiguity and ambiguity attitudes in individual decision making, as introduced by Ellsberg (1961), has been documented in static one-shot choices in many experimental studies. While the impact of lack of knowledge about probabilities is relatively well explored in static choices, less is known about its impact in a dynamic setting. In this chapter, we investigate the effect of ambiguity on behavior in a controlled laboratory experiment. This has a key advantage over studies that employ field data to test for such effects, since every empirical analysis with field data will be inevitably marred by many potential confounds. In the laboratory we are able to conduct a randomized control trial and induce an exogenous variation that is orthogonal to any other effect. The experimental results indicate that there exists a significant effect of uncertainty

and that this effect does not become insignificant over a fairly large number of repetitions of the same stopping task. Subjects in the experiment who were randomly assigned to a treatment group facing an ambiguous payoff process, invest, on average, later than subjects assigned to a control group facing a risky process. The experiment is designed in a way that under EU, those subjects facing an ambiguous payoff process should in principle stop no later than subjects facing a risky payoff process.

The experimental findings relate to several other literatures, e.g. the macroeconomic literature that is concerned with uncertainty effects à la Bloom (2009) or the literature on market microstructure or investment behavior and portfolio choice where agents are ambiguity averse (Epstein & Schneider, 2010). Finally, they may also serve as microfoundation for macroeconomic models which model representative household or firm behavior with recursive multiple-prior preferences (Ilut & Schneider, 2010).

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1 Optimal Stopping rules under Expected Utility and Regret

This chapter is based on joint work with Philipp Strack.

1.1. Introduction

We study choice behavior in the following setting: An agent observes a sequence of offers, $X = (X_1, X_2, \dots)$, which are the realizations of some stationary stochastic process. After observing the t -th offer, the agent has to decide whether to continue and thus forgo the current offer or to stop and seize it. In the former case, she observes the next offer and faces the same decision again. In the latter case, the agent's decision to stop is irreversible and she receives a net payoff $X_t - K$ from which she derives utility $u(X_t - K)$, where $K > 0$ is known a fixed. In considering which stopping rule is optimal, the agent has to weigh the immediate gains from stopping at X_t , against the loss of the option to stop at higher values in the future.

In this paper, we address the question of what do theories of dynamic behavior predict people to do in stopping problems? That is, we extend existing and also provide new results about optimal stopping rules under different preferences.

Stopping theory has been used to model many different decision-making contexts in finance, economics and statistics. Most prominently in economics, optimal stopping theory has been applied to model labor search frictions in (see *inter alia* Stigler, 1962; McCall, 1970; Rogerson et al., 2005), but also to model irreversible investment-, option pricing- and market entry decisions (McDonald & Siegel, 1986b; Jacka, 1991; Dixit & Pindyck, 1994b) or the optimal sample size for sequential hypothesis testing (Wald, 1945a). While stopping theory is widely applied, the vast majority of the theoretical (let alone experimental) literature almost exclusively considers agent with expected utility (EU) preferences.

It is a well-known result that under expected utility (EU), an agent bases her

decision solely on the current state of X_t , irrespective of the history of events that lead there. This is because an EU agent only cares about the distribution over final wealth, which is solely a function of X_t . The optimal stopping rule turns out to be simple: Stop as soon as the payoff process hits a certain reservation level. Otherwise wait. This strategy, which we call a cut-off strategy, comprises two important properties. First, it is a reservation-level strategy. That is, the agent has a unique payoff reservation level that makes it optimal for her to seize the option which is independent of the path of X_t that lead there. Second, the agent behaves time-consistently, i.e. the process is stopped *the first time* it reaches this reservation level.

Our first contribution is to show that this prediction holds under a very general notion of EU. The way we generalize the notion of EU relative to the literature, is by relaxing a standard monotonicity assumption to a single-crossing assumption on the utility function of an agent. While the latter is always satisfied for both CRRA and CARA utility functions for example, the former is not always satisfied for CARA utility.

However, the gain from weakening this assumption is more substantial. First, our resulting notion of expected utility is general enough to cover non-differentiable, non-concave utility functions. This also nests gain-loss preferences à la Kahneman & Tversky, where utility is defined over gains and losses relative to a *fixed* reference point and agents are risk-averse over gains and risk-seeking over losses. Several papers have put forward Kahneman & Tversky's prospect theory, adapted to a dynamic context, is able to better explain observed behavior in stopping problems (Barberis & Xiong, 2009; Henderson, 2012). Nonetheless, adaption of prospect theory preferences to dynamic contexts remains a delicate issue and especially in combination with probability weighting is found to run into conceptual and theoretical problems when explaining deviations from EU (see e.g. Hens & Vlcek, 2011; Ebert & Strack, 2012). Second, armed with the single-crossing assumption, we are not only able to establish that the ex-ante or minimal optimal strategy is a cut-off strategy, we also show that the subgame perfect optimal strategy is a unique cut-off strategy.

Our second contribution is to derive the optimal stopping rule for an agent with minimax regret preferences. Regret and its anticipation in a dynamic context have received considerable attention in the fields of psychology (Gilovich et al., 1998; Zeelenberg et al., 1998, 2000) and more recently neuroscience (Camille et al., 2004;

Coricelli et al., 2005, 2007) and already Shefrin & Statman (1985) conjectured it to be an important factor in the related portfolio choice problem.¹ We model a regret agent as an agent whose utility is defined over final wealth relative to what would have been *ex post* optimal (Wald, 1945b; Milnor, 1954; Stoye, 2011). In our setting this implies a dynamic reference point at the past maximum of X_t . Our first result is that even though regret preferences are time-inconsistent, the ex-ante optimal strategy of such an agent is a cut-off strategy, i.e. time-consistent. Our second result qualifies this finding in the sense that the subgame perfect optimal strategy of a regret agent is not a cut-off strategy. Loosely speaking, if we consider the possibility that agents may fail to implement their ex ante strategy, their ex post behavior is predicted to be different. Specifically, agents with EU and gain-loss preferences will behave time-consistently and *not* reconsider their behavior ex post. We show that a regret agent, however, will reconsider her plan and likely raise the ex-post cut-off to be equal to the past maximum of the payoff process.²

The chapter is organized as follows. In section 1.2, we give a brief overview over the related literature. In section 1.3, we describe the choice setting we consider. In section 1.4, we present our model of expected utility and regret and derive the optimal strategy under both preferences and discuss testable predictions. Section 3.7 finally concludes.

1.2. Related Literature

Our paper relates to several strands in the literature. The theoretical literature on optimal stopping is vast and too large to give an exhaustive overview here. Peskir & Shiryaev (2006) provide an overview over the mathematical literature on optimal stopping and free-boundary problems in discrete and continuous time, whereas Dixit & Pindyck (1994b) provide an introduction of these mathematical tools to the finance and economics literature. The theory of optimal stopping has been applied to many different contexts in economics, most prominently to job

¹Also Harry Markowitz admitted in an often-cited quote in the January issue of the magazine *Money* in 1998 that his actual portfolio choice is largely dictated by anticipated regret: "I should have computed the historical covariance of the asset classes and drawn an efficient frontier. Instead I visualized my grief if the stock market went way up and I wasn't in it – or if it went way down and I was completely in it. My intention was to minimize my future regret, so I split my [pension scheme] contributions 50/50 between bonds [risk-free assets] and equities [risky assets]."

²This provides a formal proof for the conjecture made by Gneezy (2005) about the behavior of an agent with reference point at the historical peak of the payoff process.

search models (see *inter alia* Stigler, 1962; McCall, 1970; Rogerson et al., 2005), but also to settings where firms face an irreversible investment or market-entry decision (McDonald & Siegel, 1986b; Dixit & Pindyck, 1994b). More recently, there has been growing interest in the implications of non-standard preferences for optimal stopping rules or more generally dynamic behavior. For example, Epstein & Schneider (2003); Nishimura & Ozaki (2007, 2004); Riedel (2009); Cheng & Riedel (2013) derive optimal stopping rules under ambiguity, whereas Hens & Vlcek (2011); Henderson (2012); Ebert & Strack (2012) consider settings where an agent's utility stems from realized gains and losses or the agent has prospect theory preferences à la Kahneman & Tversky (1979b).

Interest has largely focused on prospect theory preferences in a dynamic context, but there has been less interest in the impact of regret on stopping behavior. However, regret or more generally the impact of emotion on choice is likely to be an important building block in explaining prominent choice patterns such as the disposition effect. For example, Summers & Duxbury (2007) provide experimental evidence that the disposition effect does not appear if subjects are not actively trading assets, but merely experience losses without being responsible for them. Regret-based decision heuristics were introduced by Wald (1945b) and axiomatized in a static context by Milnor (1954); Loomes & Sugden (1982). Krämer & Stone (2012) consider a multi-period model with finite horizon and show that regret leads to path-dependent behavior. In our setting, the time horizon is stochastic and only almost surely finite, which takes away any end-of-game effect found by Krämer & Stone. Hayashi (2009) also considers a finite-horizon setting and focuses on dynamic consistency issues of a naïve regret agent and shows how they can be resolved.

1.3. The Setting

Time is discrete and indexed by $t \in \{0, 1, \dots\}$. The agent observes a sequence X_0, X_1, \dots of realizations of a multiplicative binomial random walk. For a given starting value $X_0 > 0$, future values of X_t are drawn according to the transition rule

$$X_{t+1} = \begin{cases} h X_t & \text{with probability } p \\ \frac{1}{h} X_t & \text{with probability } 1 - p \end{cases}.$$

We call $h > 1$ the step size and $p \in [0, 1]$ the uptick probability. We denote by $\mathcal{X} = \{h^k X_0 : k \in \mathbb{Z}\}$ the set of possible states of the process X_t . At the end of any period t there is a fixed exogenous probability $1 - \delta \in [0, 1]$ that the game ends and the agent receives a payoff of zero. We denote by $T \geq 0$ the random time the game ends. At any time $t < T$ before the game ended the agent observes the realization of the random walk X_t and decides whether to ‘continue’ or to ‘stop’.

If the agent chooses to stop in period t , she receives the value of the random walk X_t minus a constant transaction cost $K > 0$, such that her material pay-off equals

$$X_t - K .$$

Without loss of generality we assume that $K \leq X_0$.³ After an agent decided to stop, she continues to observe the realization of the process until the process jumps to zero in period T .

If the agent chooses to continue, the game ends with probability $1 - \delta$ and the agent gets a payoff of zero. With probability δ , the game does not end in period t , but period $t + 1$ starts and the agent observes the next realization of the random walk X_{t+1} .

The expected gain from stopping in period $t + 1$ instead of period t equals

$$\delta (\mathbb{E}[X_{t+1} | X_t = x] - K) - (X_t - K) = x (\delta (ph + (1 - p)h^{-1}) - 1) + (1 - \delta)K .$$

If $\delta(ph + (1 - p)h^{-1}) \geq 1$ this gain in expected payoff is positive for all $x \in \mathcal{X}$ and an expected value maximizing agent never stops. This strategy however leads to a payoff of zero and thus no optimal strategy exists. The following assumption ensures that the expected value maximizing strategy is always well defined.

Assumption 1. *To make sure the problem is well-posed for the risk-neutral agent, we assume that $\delta(ph + (1 - p)h^{-1}) < 1$.*

1.4. Theories of Dynamic Behavior

In this section, we derive our theoretical predictions under different theories of dynamic behavior. The two broad classes of preferences we consider are (i) expected utility and (ii) minimax regret preferences.

³Otherwise it is always optimal for the agent to at least wait until he reaches K or the game ends.

For each class, we describe the underlying model assumptions and derive key properties about predicted behavior. It turns out that both theories predict that agents will use simple strategies, i.e. cut-off strategies. A cut-off strategy, is a strategy which satisfies the following definition

Definition 1 (Cut-off Strategy). *The cut-off strategy $\tau(b)$ prescribes that the agent stops at time t if the value of the process X_t exceeds the cut-off b and continues otherwise. That is*

$$\tau(b) = \min\{t \geq 0 : X_t \geq b\} . \quad (1.1)$$

It is worth to stress that a cut-off strategy comprises *two* important features. First, it is a reservation level strategy, i.e. it says that there exists a unique level at which it is optimal to stop for the agent. Second, it postulates that the process is stopped the *first* time the process reaches this level. This second property essentially is a time-consistency property. A strategy that satisfies both, the reservation-level property and the time-consistency property is what we call a cut-off strategy. A violation of the second property, i.e. stopping at a point X_t with $X_t < \max_{s \leq t} X_s$, indicates time-inconsistent behavior and cannot be rationalized by expected utility. What is surprising, however, is that it is also not rationalizable by certain classes of path-dependent preferences. We demonstrate this for the case of regret preferences below.

1.4.1. Expected Utility

An expected utility agent evaluates outcomes according to the strictly increasing (and not necessarily concave) utility function $u : [-K, \infty) \rightarrow \mathbb{R}$. Denote by $\mathbf{1}_A$ the indicator function that takes the value one on the event A and zero otherwise. The EU agent then chooses the stopping time τ that maximizes

$$\mathbb{E} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K) + \mathbf{1}_{\{\tau \geq T\}} u(0) \mid X_0 = x] . \quad (1.2)$$

Because preferences over stopping times are invariant under additive translations of the utility u , we can without loss of generality assume $u(0) = 0$. To shorten notation we denote conditional expectations by

$$\mathbb{E}_{t,x} [\cdot] = \mathbb{E} [\cdot \mid X_t = x, T > t]$$

$\mathbb{E}_x[\cdot] = \mathbb{E}_{0,x}[\cdot]$ and conditional probabilities by $\mathbb{P}_{t,x}[\cdot] = \mathbb{P}[\cdot | X_t = x, T > t]$ and $\mathbb{P}_x[\cdot] = \mathbb{P}_{0,x}[\cdot]$. Moreover, we introduce $V(\tau, x)$ as the expected utility of the agent when she uses the strategy τ and the initial value of the process is x

$$V(\tau, x) = \mathbb{E}_{t,x} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K)] .$$

Let us denote by $V^* : \mathcal{X} \rightarrow \mathbb{R}$ the value of the agent when he uses the optimal strategy

$$V^*(x) = \sup_{\tau} V(\tau, x) .$$

The following lemma proven in the Appendix establishes a probability theoretic result that will be useful to derive the optimal strategy.

Lemma 1 (Probability to Stop before the Deadline). *When using the cut-off strategy $\tau(b)$ as a continuation strategy at a given level $X_t = x$, the probability of stopping before the game ends, $\tau(b) < T$, is given by*

$$\mathbb{P}_{t,x} [\tau(b) < T] = \begin{cases} \left(\frac{x}{b}\right)^\alpha & \text{for all } b \geq x \\ 1 & \text{else} \end{cases} ,$$

where α is given by $\alpha = \frac{1}{\log(h)} \log \left(\frac{1}{2p\delta} + \sqrt{\frac{1}{4p^2\delta^2} - \frac{1-p}{p}} \right) > 1$.

As a consequence of Lemma 1 the expected utility from using the cut-off strategy $\tau(b)$ as a continuation strategy from $x \leq b$, equals

$$\begin{aligned} V(\tau(b), x) &= \mathbb{E}_{t,x} [\mathbf{1}_{\{\tau(b) < T\}} u(X_{\tau(b)} - K)] = \mathbb{P}_{t,x} [\tau(b) < T] u(b - K) \\ &= \left(\frac{x}{b}\right)^\alpha u(b - K) . \end{aligned}$$

At any point $x > b$, the cut-off strategy $\tau(b)$ stops immediately and therefore

$$V(\tau(b), x) = \begin{cases} \left(\frac{x}{b}\right)^\alpha u(b - K) & \text{for } x \leq b \\ u(x - K) & \text{for } x > b \end{cases} . \quad (1.3)$$

If the agent decides to stop at a point x his payoff equals $u(x - K)$ if she decides to continue until either the process reached xh or the game ended he gets an expected payoff of

$$V(\tau(xh), x) = h^{-\alpha} u(xh - K) .$$

Definition 2. We denote by $\Gamma : \mathcal{X} \rightarrow \mathbb{R}$ the expected gain from waiting until the process reached xh instead of stopping at x

$$\Gamma(x) = h^{-\alpha}u(xh - K) - u(x - K).$$

Γ describes the expected gain from waiting until the process makes one uptick. The following lemma shows that the gain from any other cut-off strategy can be expressed in terms of Γ .

Lemma 2 (Expected Payoff of a Cut-off Strategy). *The expected gain from using the cut-off strategy $\tau(xh^n)$ instead of stopping at x is given by*

$$V(\tau(xh^n), x) = u(x - K) + \sum_{j=1}^n h^{-(j-1)\alpha} \Gamma(xh^j).$$

Proof. We show the result inductively using the fact that once the agent reaches xh^{n-1} his continuation value is given by the expected value of waiting for one uptick

$$\begin{aligned} V(\tau(xh^n), x) &= \mathbb{E}_x [\mathbf{1}_{\{\tau(xh^n) < T\}} u(xh^n - K)] = \mathbb{E}_x [\mathbf{1}_{\{\tau(xh^{n-1}) < T\}} V(\tau(xh^n), xh^{n-1})] \\ &= \mathbb{E}_x [\mathbf{1}_{\{\tau(xh^{n-1}) < T\}} (\Gamma(xh^{n-1}) + u(xh^{n-1} - K))] \\ &= V(\tau(xh^{n-1}), x) + \mathbb{P}_x [\tau(xh^{n-1}) < T] \Gamma(xh^{n-1}) \\ &= V(\tau(xh^{n-1}), x) + h^{-(n-1)\alpha} \Gamma(xh^{n-1}). \end{aligned}$$

The result follows inductively in combination with the fact that $V(\tau(x), x) = u(x - K)$. \square

Define the point $b^u \in \mathcal{X}$ as the smallest point such that it is not profitable to wait until the process reaches $b^u h$, i.e.

$$b^u = \min\{x \in \mathcal{X} : \Gamma(x) \leq 0\}.$$

By definition of b^u it is never optimal to stop below b^u . If $\Gamma(b^u) = 0$ the agent is indifferent between stopping at b^u and waiting for one more uptick. Then, $\tau(b^u)$ can not be the unique optimal strategy. As this case is non-generic under random small perturbations of u we assume through the paper that $\Gamma(b^u) \neq 0$.

Definition 3 (Expected Change). *For every function $w : \mathcal{X} \rightarrow \mathbb{R}$, we denote by*

$\mathcal{L}w : \mathcal{X} \rightarrow \mathbb{R}$ the expected change in w from period t to period $t + 1$, conditional on being at x

$$\begin{aligned}\mathcal{L}w(x) &= \mathbb{E}_{t,x} [\mathbf{1}_{\{t+1 < T\}} w(X_{t+1} - K) - w(X_t - K)] \\ &= \delta (p w(xh - K) + (1 - p) w(xh^{-1} - K)) - w(x - K) .\end{aligned}$$

The following assumption ensures that the optimal strategy always stops above b^u .

Assumption 2 (Single Crossing). *The expected change in utility $\mathcal{L}u(x - K)$ is negative for all $x > b^u$.*

Assumption 2 ensures that stopping immediately is better than continuing and stopping in the next period for all $x > b^u$. As the next Lemma shows Assumption 2 is a necessary condition for optimal strategies to be cut-off strategies

Lemma 3. *If Assumption 2 is violated and u is concave no optimal strategy is a cut-off strategy.*

It can be shown that if u is not concave and Assumption 2 is violated at least one optimal strategy is not a cut-off strategy. We say that an agent has constant absolute risk-aversion if $u(x) = -\frac{1}{\theta} \exp(-\theta x)$ for some $\theta \geq 0$ and has constant relative risk-aversion if $u(x) = \frac{1}{\theta} ((x + K)^\theta - K^\theta)$ for some $\theta \in (0, \alpha)$.⁴ The following Lemma is proven in the Appendix.

Lemma 4. *Assumption 2 is satisfied if u has constant absolute or relative risk-aversion.*

As the next Proposition shows Assumption 2 is sufficient to ensure that stopping is better than any continuation strategy for all $x \geq b^u$.

Proposition 1 (The Optimal Strategy). *The unique subgame perfect optimal strategy continues for all values $x < b^u$ and stops for all values $x \geq b^u$.*

Proof. $\tau(b^u)$ is an optimal strategy:

In the first step we prove that stopping above b^u is an optimal strategy. To shorten

⁴To ensure the utility of negative outcomes is well defined we look at constant relative risk-aversion relative to the wealth level $(x + K)$.

notation let us denote by $W : \mathcal{X} \rightarrow \mathbb{R}$ the continuation value from using the cut-off strategy $\tau(b)$ derived in (1.3)

$$W(x) = V(\tau(b^u), x) = \begin{cases} \left(\frac{x}{b^u}\right)^\alpha u(b^u - K) & \text{for } x \leq b^u \\ u(x - K) & \text{for } x > b^u \end{cases}.$$

By the dynamic programming principle (cf. Peskir & Shiryaev, 2006, Theorem 1.11), $\tau(b^u)$ is an optimal strategy if and only if the function $W(x)$ satisfies the dynamic programming equation for all $x \in \mathcal{X}$

$$\max\{\mathcal{L}W(x), u(x - K) - W(x)\} = 0. \quad (1.4)$$

We have that $W(x) = u(x - K)$ for all $x \geq b^u$. Hence, $\mathcal{L}W(x) = \mathcal{L}u(x) < 0$ for all $x > b^u$ and (1.4) is satisfied for all $x > b^u$. Let $n = \frac{\log(b^u/x)}{\log(h)}$, by Lemma 2 and the definition of b^u we have that for all $x < b^u$

$$u(x - K) - W(x) = - \sum_{j=1}^n h^{-(j-1)\alpha} \Gamma(xh^j) < 0.$$

For all $x < b^u$ it holds that $\mathbb{E}_{t,x}[\mathbf{1}_{\{t+1 < T\}} W(X_{t+1})] = W(x)$, thus $\mathcal{L}W(x) = 0$ for all $x < b^u$, and hence (1.4) is satisfied for all $x < b^u$. It remains to verify that (1.4) is satisfied for $x = b^u$. By definition $W(b^u) = u(b^u - K)$ and thus, it remains to prove that $\mathcal{L}W(b^u) \leq 0$

$$\begin{aligned} \mathcal{L}W(b^u) &= \mathbb{E}_{t,b^u} [\mathbf{1}_{\{T > t+1\}} W(X_{t+1}) - W(X_t)] \\ &= \delta \left[p W(b^u h) + (1 - p) W(b^u h^{-1}) \right] - u(b^u - K) \\ &= \delta \left[p u(b^u h - K) + (1 - p) \left(\frac{b^u h^{-1}}{b^u} \right)^\alpha u(b^u - K) \right] - u(b^u - K) \\ &= u(b^u - K) \left[\delta \left(p \frac{u(b^u h - K)}{u(b^u - K)} + (1 - p) h^{-\alpha} \right) - 1 \right]. \end{aligned}$$

By definition of b^u and as $\Gamma(b^u) \neq 0$ we have $u(b^u h - K)/u(b^u - K) < h^\alpha$. As h is the larger solution to the equation $\delta(ph^\alpha + (1 - p)h^{-\alpha}) = 1$ the expected change of W at b^* is negative

$$\mathcal{L}W(b^u) < u(b^u - K) (\delta [p h^\alpha + (1 - p) h^{-\alpha}] - 1) = 0.$$

$\tau(b^u)$ is the unique optimal strategy:

By Definition of b^u it is never optimal to stop at $x < b^u$. As we have shown that $V^* = W$ and $\mathcal{L}W(x) = \mathcal{L}u(x - K) < 0$ for all $x > b^u$ and thus it is never optimal to continue at $x > b^u$. As shown above $\mathcal{L}W(b^u) < 0$ and hence it is not optimal to continue at b^u . \square

Proposition 1 did not require the utility function u to be differentiable or concave as long as Assumption 2 is satisfied. It therefore covers cases where u has a kink at a reference point r . Where this reference point lies is immaterial to our results, as long as r is determined *a priori* and constant.

As no concavity of u is required Proposition 1 furthermore covers cases of S-shaped utility as in Kahneman & Tversky (1979b), i.e. risk-seeking behavior below and risk-averse behavior above the reference point.⁵

When the reference point r is the *a priori* expected utility from stopping the process then this may be viewed as a model of disappointment à la Loomes & Sugden (1986). It then follows from Proposition 1 that a model of elation or disappointment does not predict path-dependent behavior in our setting. Interestingly, experimental evidence seems to support this prediction. For example, Summers & Duxbury (2007) find that in an experiment where subjects do not actively trade fictitious assets, the disposition effect does not appear, while it does so when subjects had to actively choose their portfolio. They conclude that regret and self-blame as opposed to disappointment, which lacks the self-blame component, is a key building block in explaining the disposition effect. Our theoretical model provides a rigorous argument for this finding.

1.4.2. Regret Preferences

In this section we examine the model predictions if the agent experiences regret. For a regret agent, the utility associated with the consequence of his action, is not solely a function of final wealth, but the difference between final wealth and the *ex post* optimal outcome. If the action chosen by the agent is *ex post* suboptimal, the agent feels regret. This makes choice context dependent, because the expected utility associated with a given act, depends on what are the counterfactual outcomes of the remaining acts available to the agent. In our setting, it is

⁵For a detailed discussion of the stopping behaviour of prospect theory agent with probability distortion and naivite (Ebert & Strack, 2012), with probability distortion and commitment (Xu & Zhou, 2013) or without probability distortion see Henderson (2012).

always *ex-post* optimal for the agent to stop when the process was at its maximum. Therefore regret preferences are preferences where the historical peak of the process is the reference point. The objective of the agent is then to minimize the expected regret.

To model regret aversion in our setting, we assume that the intensity with which the agent feels regret is linear in the utility difference between his strategy and the strategy that turns out to be *ex-post* optimal. In our setting there are generally two possibilities: regret relative to past and future decisions or regret relative only to past decisions.

Regret over Past and Future Decisions

Several authors argue that the anticipation of future regret affects choices in the present (see i.e. Loomes & Sugden, 1982). More precisely, because the agent does observe the process even after she stopped, she might also anticipate to feel regret relative to the maximum attained not only prior to stopping, but over the whole time horizon until T .

Let us denote by $S_t = \max_{r \leq t} X_r$ the maximal value of the process prior to time t . Clearly, the ex-post optimal decision for the agent is to stop when the process reaches its maximal value S_T , which would have given her a utility of $\max_t u(X_t - K) = u(S_T - K)$.⁶ The regret experienced by the agent is linear in her loss of utility due to taking a suboptimal decision

$$u(S_\tau - K) - \mathbf{1}_{\{\tau < T\}} u(X_\tau - K).$$

Note, that the agent enjoys the benefits of stopping $u(X_\tau - K)$ only if she stops before the deadline $\tau < T$, while she suffers the regret also if she does not stop before T . Thus, after a history such that the value of the process equals x and its past maximum equals s in period t the regret value associated with the stopping strategy $\tau \geq t$ equals

$$\begin{aligned} V(\tau, x, s) &= (1 - \kappa) \mathbb{E}_{t,x,s} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K)] - \kappa \mathbb{E}_{t,x,s} [u(S_T - K) - \mathbf{1}_{\{\tau < T\}} u(X_\tau - K)] \\ &= \mathbb{E}_{t,x,s} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K)] - \kappa \mathbb{E}_{t,x,s} [u(S_T - K)] , \end{aligned} \quad (1.5)$$

where $\kappa \in [0, 1)$ denotes the intensity of regret. Regret preferences contain ex-

⁶Note, that we assumed that the process starts above K and thus stopping at the maximal value $S_T \geq K$ is always better than not stopping at all.

pected utility preferences for $\kappa = 0$.

However, the case where the agent feels regret relative to past and future decisions does not yield behaviorally different predictions from expected utility. To see this, note that the expected regret $\mathbb{E}[u(S_T - K)]$ is independent from the agent's stopping strategy τ and thus an agent who exhibits regret over past and future decisions behaves exactly as the corresponding EU agent.

Proposition 2. *The cut-off strategy $\tau(b^u)$ is the unique optimal continuation strategy for the agent experiencing regret over past and future decisions. That is, $\tau(b^u)$ maximizes the regret functional defined in Equation (1.5) after every history.*

Thus, there is no way to distinguish regret over past and future decisions from EU. We therefore consider only regret over past decisions below.

Regret Only over Past Decisions

If the agent feels regret only relative to past decisions, the ex-post optimal decision for the agent is to stop at the time t before τ when the process reached its maximal value. This strategy would have given her a utility of $\max_{t \leq \tau} u(X_t - K) = u(S_\tau - K)$. Thus, the regret functional equals

$$V^r(\tau) = \mathbb{E} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K)] - \kappa \mathbb{E} [u(S_\tau - K)] . \quad (1.6)$$

As in the expected utility case we denote the value from using the continuation strategy τ at the point $X_t = x$ and the past maximum equals $S_t = s$ by $V^r(\tau, x, s)$.

An important feature of regret preferences over past decisions is their history-dependence. At first glance such preferences seem to have the potential to rationalize behavior that under EU would have been classified as time-inconsistent. In an asset-selling setting the incentive to sell the asset, is higher the lower the historical peak relative to the current price. Given that the reference or aspiration level of an agent changes with the history of the process, one might believe that, because the agent is reluctant to realize a loss, she adopts a path-dependent strategy. The results below, however, show that if the agent always behaves optimally this is *not* the case.

We begin by establishing that the regret agent never stops after the agent maximizing expected utility.

Lemma 5. *It is optimal for the regret agent to stop at all points $x \geq b^u$.*

Proof. We first show that the regret agent always stops above the expected utility cut-off b^u . Note, that by definition of b^u any strategy that continues at a point $x \geq b^u$ yields a change in utility which is negative in expectation

$$\sup_{\tau > t} \mathbb{E}_{t,x} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K)] < u(x - K).$$

Furthermore, the maximum only increases over time and thus regret increases, i.e. $\mathbb{E}_{t,x,s} [u(S_\tau - K)] > u(s - K)$ for all stopping times $\tau > t$. Hence, it is optimal to stop for all values $x \geq b^u$

$$\sup_{\tau > t} \mathbb{E}_{t,x,s} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K) - \kappa u(S_\tau - K)] < u(x - K) - \kappa u(s - K). \quad \square$$

Intuitively, regret can only make it less attractive to continue, as continuing always entails the risk of increased regret. As continuing above b^u is not optimal without regret it can never be optimal with regret. In the next step we show that below the expected utility cut-off is never optimal for the regret agent to stop below the past maximum.

Lemma 6. *It is never optimal for the regret agent to stop when $X_t < S_t \leq b^u$.*

Proof. Let $X_t = x < s = S_t \leq b^u$. By Lemma 2 the expected change in utility from the cut-off strategy $\tau(s)$ that only stops once the past maximum is reached is positive

$$\mathbb{E}_{t,x} [\mathbf{1}_{\{\tau(s) < T\}} u(X_{\tau(s)} - K)] > u(x - K).$$

As the strategy $\tau(s)$ never stops above the past maximum s it follows that the regret never increases $u(S_{\tau(s)} - K) = u(s - K)$. Thus, it is always better to wait until the process is back at its past maximum s than stopping at a value $x < s$

$$\mathbb{E}_{t,x,s} [\mathbf{1}_{\{\tau(s) < T\}} u(X_{\tau(s)} - K) - \kappa u(S_{\tau(s)} - K)] > u(x - K) - \kappa u(s - K). \quad \square$$

Intuitively as regret is sunk and does not change until the process reaches his past maximum again it can never be optimal to stop the process below its past maximum. Note, that as a consequence of Lemma 6 the agent never experiences regret when stopping below b^u .

We define $\Gamma^r : \mathcal{X} \rightarrow \mathbb{R}$ as the expected change in value from waiting until the

process reaches xh instead of stopping at $x = X_t = S_t$:

$$\begin{aligned}
\Gamma^r(x) &= (\mathbb{P}_x[\tau(xh) < T] (1 - \kappa)u(xh - K) - \mathbb{P}_x[\tau(xh) \geq T] \kappa u(x - K)) \\
&\quad - (1 - \kappa)u(x - K) \\
&= h^{-\alpha}(1 - \kappa)u(xh - K) - \kappa(1 - h^{-\alpha})u(x - K) - (1 - \kappa)u(x - K) \\
&= h^{-\alpha}(1 - \kappa)u(xh - K) - (1 - \kappa h^{-\alpha})u(x - K).
\end{aligned}$$

Define the cut-off b^r by

$$b^r = \min\{x \in \mathcal{X} : \Gamma^r(x) \leq 0\}. \quad (1.7)$$

As in the expected utility case we assume that $\Gamma^r(b^r) \neq 0$. Furthermore, we assume that the expected gain in payoff from waiting for an up-tick Γ^r changes its sign once.

Assumption 3. *We assume that $\Gamma^r(x) < 0$ for all $x \geq b^r$.*

This is a weak assumption and the next Lemma establishes that it is satisfied for all risk-averse agents.

Lemma 7. *If u is concave than Assumption 3 is satisfied.*

Proof. First, we establish existence of b^r . For that we need to show that Γ^r crosses zero at least once. The change of Γ^r in x equals

$$\begin{aligned}
\Gamma^r(xh) - \Gamma^r(x) &= (h^{-\alpha}(1 - \kappa)u(xh^2 - K) - (1 - \kappa h^{-\alpha})u(xh - K)) \\
&\quad - (h^{-\alpha}(1 - \kappa)u(xh - K) - (1 - \kappa h^{-\alpha})u(x - K)) \\
&= (1 - \kappa) h^{-\alpha} (u(xh^2 - K) - u(xh - K)) \\
&\quad - (1 - \kappa h^{-\alpha}) (u(xh - K) - u(x - K))
\end{aligned} \quad (1.8)$$

As u is concave, we know that

$$\frac{u(xh - K) - u(x - K)}{x(h - 1)} \geq \frac{u(xh^2 - K) - u(xh - K)}{xh(h - 1)}$$

Multiplying by both sides $x(h - 1)$ and as $\alpha > 1$ it follows

$$\begin{aligned}
u(xh - K) - u(x - K) &\geq h^{-1} (u(xh^2 - K) - u(xh - K)) \\
&> h^{-\alpha} (u(xh^2 - K) - u(xh - K)),
\end{aligned}$$

As $1 - \kappa < 1 - \kappa h^{-\alpha}$ Equation (1.8) is negative. Consequently, Equation Γ is strictly decreasing and changes sign at most once. \square

The next proposition shows that it is optimal to stop if and only if the past maximum of the process S_t is at least b^r and the process is at its maximal value $X_t = S_t$ or the process x_t is above the expected utility cut-off b^u .

Proposition 3. *The optimal strategy stops if and only if $X_t \geq b^r$ and $X_t \geq \min\{b^u, S_t\}$.*

Proof. We already establishes that is optimal to stop $X_t \geq b^u$ and optimal to continue for $X_t < S_t \leq b^u$. By definition of b^r it is optimal to wait for an up-tick at $X_t = S_t < b^r$. It remains to show that it is optimal to stop for $X_t = S_t \geq b^r$.

We prove this result by induction. As show it is strictly optimal to stop at $X_t = S_t = b^u$. Let $X_t \geq b^r$ and suppose it is strictly optimal to stop once the process reaches $X_t h$. Then, the change in payoff from waiting for this uptick is given by

$$V^r(x) - (1 - \kappa)u(x - K) = \Gamma^r(x),$$

which is negative by definition of b^r . Hence, it is strictly optimal to stop at X_t . \square

We have plotted the subgame perfect optimal stopping strategy of the regret agent in Figure 1.1. For $s < b^r$, the agent's optimal continuation strategy is the cut-off strategy $\tau(b^r)$, especially it is independent of s . The optimal continuation strategy changes if the agent misses to fulfill her initial plan, i.e. to stop at b^r finds herself in a history where s exceeds b^r . The optimal continuation strategy now prescribes the agent to wait for the process to return to s .

Consider a path where the process reaches b^r a second time, i.e. $X_t = b^r$ and $s > b^r$. Under EU, the situation has not changed relative to the first visit, because the running maximum s is immaterial to the EU agent. For the regret agent, this is different: Compared to the situation where $s = x = b^r$, the regret associated increased. The higher regret also enters the continuation value, but to lesser extent, because continuation includes the prospect of making up for the current regret. The prospect of making up for the current regret motivates the agent to continue until the process reach s again.⁷

⁷This behavior resembles the gambling for resurrection described in the finance literature. While gambling for resurrection in this literature occurs, because the agent is insured against losses, in the regret model the agent continues because further losses do not translate into higher regret.

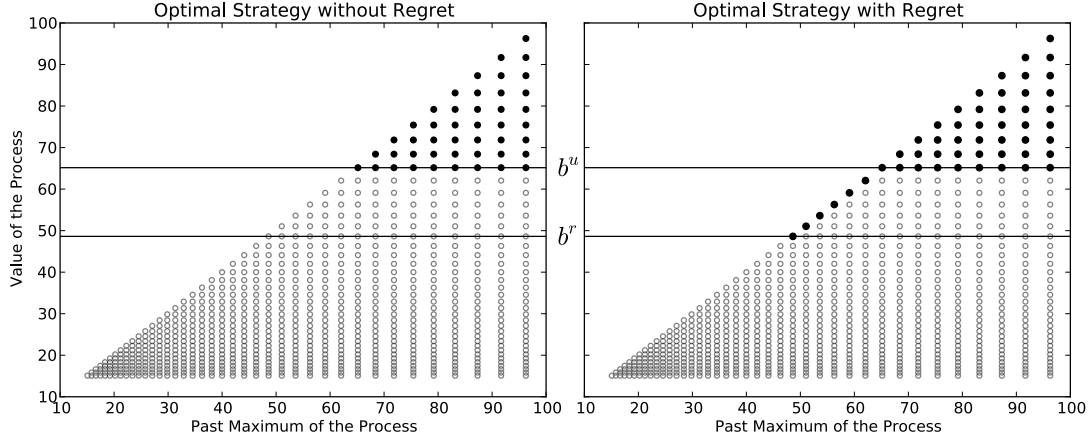


Figure 1.1.: This shows the subgame perfect cut-off of an agent without regret on the left and with regret on the right. Empty dots mark the points where the agent continues and filled large dots show where it is optimal for the agent to stop.

Note, that while the optimal strategy for the EU and the regret agent both lead to the **same** observed behavior. As the regret agent always stops at b^r the realized (observed) stopping time will be equal to the realized stopping time resulting from the cut-off strategy $\tau(b^r)$.

Proposition 4. *Let τ^r be the strategy which is optimal for the regret agent. We have that $\tau^r = \tau(b^r)$ almost surely.*

Proof. This follows directly from Proposition 3 under the assumption that the agent almost surely does not commit mistakes. \square

Hence, regret is observationally equivalent to expected utility. This equivalence relies on the premise that the agent makes no mistakes. If the agent makes mistakes, behavioral differences arise and regret and expected utility can be told apart from observed decisions. We will use laboratory data in combination with this identification strategy in the following chapter to identify regret preferences.

1.5. Conclusion

Our results show that in the classical setting, where an agent has the option to stop a binomial random walk a very robust prediction about behavior arises: People

are expected to use cut-off strategies. That is, they are predicted to (i) stop the process at a unique reservation level that depends on their preferences and (ii) to never stop at a point they chose to continue before. We show that is indeed a very robust prediction about behavior which holds not only under EU, but also for gain-loss preferences and also for an agent who experiences regret. While this is by far not a new result under EU, we demonstrated that it holds under more general circumstances than previously known. Hence, from a theoretical perspective, we would expect subjects in a controlled environment to play roughly the same reservation level across different repetitions of the same stopping task and to behave time-consistently within rounds. We test this prediction in a companion experiment in the next chapter.

2 Too proud to stop: Stopping behavior in the laboratory

This chapter is based on joint work with Philipp Strack.

2.1. Introduction

Many important economic decisions are modeled as optimal stopping problems, where an agent has to trade the immediate gains from stopping against the loss of the option to act tomorrow. While stopping theory has been widely applied throughout economics, there is little empirical, let alone experimental, evidence on whether theoretical models of optimal stopping describe actual choices. Given that stopping theory is widely applied, however, it appears important to test key predictions of it and gauge in how far it is able to predict actual behavior. This chapter puts forward a direct test in form of a controlled laboratory experiment to shine more light on this issue.

In the given setting we have shown earlier that the optimal stopping rule is simple: The optimal strategy is to stop the first time the payoff process crosses a certain threshold level. Hence, the optimal strategy has two important properties: (i) it is a reservation-level strategy, i.e. the agent has a unique payoff reservation level that makes it optimal for her to seize the option, and (ii) the agent behaves time-consistent, i.e. the process is stopped *the first time* it reaches this reservation level.

While the theoretical literature on optimal stopping theory is vast (see e.g. Peskir & Shiryaev, 2006, for an overview) few experimental studies on choice behavior in optimal stopping problems exist. Most of these studies solely consider the expected utility benchmark for behavior and are solely concerned with inspecting whether subjects' reservation levels fall short of or exceed the risk-neutral level. For example, it is inspected in a stylized job-search setting whether

subjects search too much or too little before taking an offer, or whether they take an offer at a value x_i that is above or below the risk-neutral optimal cut-off. A common conclusion of nearly all of these studies is that expected utility describes behavior well. For example, Rapoport & Tversky (1970) write: "*The results of the present study suggest that the optimal model provides a reasonably good account of the behavior of the subjects.*" (p.112) and more recently Oprea, Friedman & Anderson (2009) for example conclude that: "*(...) behavior approaches optimality in all treatments.*" (p. 1104).

However, if optimal stopping theory is to describe actual behavior it is arguably even more important that choices strictly adhere to the second property of the optimal strategy than to the first. Stopping a process at a value that has previously been reached cannot be reconciled with *any* time-consistent preference of the expected-utility type and even time-inconsistent preferences such as minimax regret. Finding that such behavior is abundant would therefore cast doubt on the claim that expected utility can provide an adequate description of actual behavior. To the authors' best knowledge, there is no paper that systematically inspects deviations along this dimension.¹ Hence, a crucial deviation from the prediction of expected utility may have gone undetected in earlier papers.

And indeed, the data we obtained from a choice experiment in the laboratory confirms that time-inconsistent behavior is abundant. In our sample (i) subjects generally show no tendency to have a unique reservation level across rounds and show little to no convergence towards such a constant level, and (ii) subjects in the laboratory make time-*inconsistent* choices in 75% of all cases. This, on the one hand, is a strong albeit discouraging result in the given setting, because it means that 75% of the observed stopping choices cannot be reconciled with any of the deterministic models we have analyzed before.

On the other hand, visual inspection of our data already suggests that deterministic models of choice are likely to be descriptively deficient. In order to account for the possibility that large part of the variation and time-inconsistency could be due to random choice errors, we then use our closed-form expressions for the continuation and stopping value of an agent to fit a dynamic binary choice model to the data. Such an empirical model is not only interesting because it is able to capture noise in the data, but also because in a model with random choice errors, the regret model becomes testable. Hence, given the stopping decisions of

¹A notable exception is Gneezy (2005), who makes a first step in this direction.

subjects, we are able to use the likelihood principle to assess whether observed decisions lend more support to a model of random expected utility or a model of random regret. Recall that the behavioral difference between regret and expected utility arises in a model where agent may unexpectedly fail to implement their ex ante plan. That is, when a regret agent reaches a point above her ex ante optimal cut-off, but below the running maximum of the process, she will not want to stop, but wait for the process to return to its previous maximum and stop then. In fact, this is a choice pattern we observe to be abundant in our data. Fitting a structural econometric model to the tick-level data on stopping decisions, we find that a model with regret aversion explains the data significantly better than a model without regret. This result also relates to the finding by Oprea et al. (2009), that subjects seem adjust their reservation levels in a given round in response to forgone earnings in previous rounds. While Oprea et al. document a form of inter-round regret, we find that also intra-round regret affects the reservation level of subjects.

The paper is structured as follows: In section 1.2 we briefly review the existing literature. In section 1.3 we introduce the general setting and our notation. Section 1.4 provides the main theoretical results. It only provides a few proofs that we feel to be crucial for reading. All other technical details can be found in the appendix. In section 2.4 we present the experimental results and an empirical analysis with respect to the model predictions.

2.2. Related Literature

Compared to the theoretical literature, the experimental literature on optimal stopping is relatively small. The first experimental papers due to Amnon Rapoport and co-authors, contrast the theoretical predictions from sequential search models with individual behavior in the laboratory (see *inter alia* Rapoport & Tversky, 1966; Kahan, Rapoport & Jones, 1967; Rapoport & Tversky, 1970; Seale & Rapoport, 1997, and references therein). These experiments test theoretical predictions made by models of sequential search. Caplin, Dean & Daniel (2011) inspect a static search model with recall, where subjects had unlimited time to search among a set of alternatives displayed on a screen. In a similar vein, there is a strand of experimental literature putting a focus on testing implication for the particular class of job search models in the laboratory along various dimensions, e.g. Schotter & Braunstein (1981); Cox & Oaxaca (1989, 1992, 2000) and Brown,

Table 2.1.: Parameters for the binomial random walk in the experiment.

Cost K	Stepsize h	Uptick prob. p	Exp. prob. $1 - \delta$
40	1.06	52 %	0.7 %

Flinn & Schotter (2011).

Oprea et al. (2009) choose an experimental design, aimed at replicating the investment setting motivated in Dixit & Pindyck (1994b). In their paper, subjects observe a geometric binomial random walk and have the option to earn the current value of the random walk less some fixed cost, or forgo it in favor of future values. We replicate their setup in this paper. They find that subjects approximate the risk-neutral optimal strategy surprisingly well. While in three out of four treatments with different parameters for the evolution of the random walk, subjects stop too early, stopping decisions are nearly optimal in one treatment. Moreover, Oprea et al. show that subjects adjust their reservation levels in response to forgone earnings, i.e. regret associated with their stopping decision in the current round, leads them to reconsider their strategy in the coming rounds.

2.3. A laboratory experiment

In order to investigate individuals' behavior and to test our predictions about behavior, we implemented exactly the theoretical setting described in the previous section in the laboratory. Testing individual behavior in a controlled laboratory setting has several advantages over using field data. First, we can ensure that the environment in which subjects make their decisions is truly stationary, i.e. the probability law driving the process is known and time-homogeneous. Second, we also have full discretion over the payoff-relevant state variables of a subject. With field data, we can never be entirely sure to observe all relevant state variables an individual integrates into her decision-making process. In the laboratory, we have full discretion over all payoff-relevant state variables. Especially for testing the time-consistency property, both points are crucial. The experiment was conducted as a computer-based experiment at the laboratory of the Technical University Berlin (TU) and the WZB Berlin. The experimental software was programmed using Java and Python and ran in a browser. We ran two sessions, each session with 22 students that were randomly recruited from the ORSEE pool

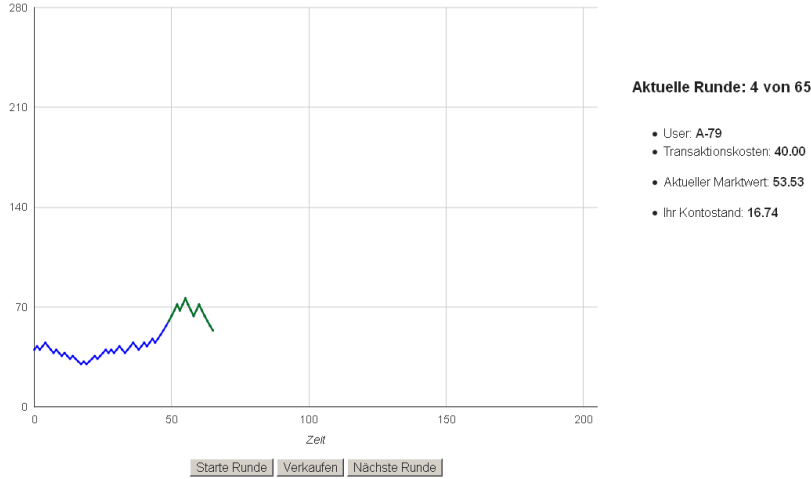


Figure 2.1.: The main experimental screen (in German).

of the TU and WZB.² Each of the 44 subjects in our sample, played 65 rounds for pay. For each round played by subjects, we either observe a stopping decision or that the process jumped to zero before a subjects decided to stop. The latter is true for about 56%, i.e. 1581 out of the total 2860, in our sample.

Before the actual experiment, subjects received a four-page instruction explaining the upcoming experiment. In the instructions, we framed the optimal stopping task as an asset-selling problem. Subjects were explained that they own a fictitious stock and that they have the opportunity, but not the obligation, to sell it. The instructions then explained in detail the setting discussed in section 1.3 and the meaning of the relevant parameters, e.g. the uptick probability p and the step size h . The actual values for the parameters were given to subjects and are listed in Table 2.1. Subjects were hence fully informed about how processes were generated.³

After subjects finished reading the instructions, they were prompted to login and begin the actual experiment. The experiment consisted of 65 rounds in which subjects had the option to sell their stock. In each round, subjects observed the path of the market price in a diagram (see Figure 2.1). At the beginning of each

²See: <https://experimente.wzb.eu/>

³We convinced ourselves that subjects had indeed understood (i) how payoffs are computed, (ii) that the increments of the process are iid and (iii) what is the risk that a round ends before the next period, through a questionnaire with control questions that subjects had to answer prior to the experiment (see appendix). 95% of the time subjects answered our control questions correctly.

round, the computer loaded the screen with an empty diagram. At the bottom of the screen there were three buttons available. Upon pressing the left-hand button, labeled 'Start round', subjects started a given round.⁴ That triggered the market price to be displayed as a jagged blue line until the jump to zero in period T . Each second there were two ticks of the price process. Additionally, subjects were displayed several other details about a round in a panel to the right of the diagram. As soon as a round was started, the middle button, labeled 'Sell', became active. Pressing this button, a subject sold the stock at the current value of the price process. Future values of the process were then displayed in green as to visualize that selling had occurred. The right-hand button, which was inactive until the jump to zero, gave subjects the opportunity to move to the next round.⁵ The paths of the 65 random walks were generated prior to the experiment and were the same for all subjects. However, based on an individual login printed on the instructions, the order in which the set of 65 paths was shown to subjects was shuffled randomly.

At the end of the experiment one round was randomly selected with equal probability to determine a subject's payoff. Subjects were informed about which round was drawn on a final screen that listed their performance in each round played together with their final payoff. Subjects earned 0.15 times the number of points they had obtained in the round that was drawn by the computer plus 10 Euros show-up fee.⁶ The average duration of the overall experiment was 80 minutes, and the mean earnings for subjects was 12.30 Euros (median=10 Euros), where the minimum and the maximum payment were 10.00 Euros and 19.00 Euros respectively.

2.4. Experimental results

Following the literature, we call the value X_τ at which the agent decided to stop her reservation level. As shown in Proposition 1 and 3 an agent maximizing expected utility or minimizing regret should **not** vary her reservation level between rounds. We thus first inspect to which extent subjects do have a constant reservation

⁴The remaining two buttons were disabled before the round was started.

⁵We could have given subjects the option to skip to the next round immediately after the stopping decision. This, however, may provide incentives to impatient subjects to stop early and reduce lab time.

⁶We rounded to the nearest Euro.

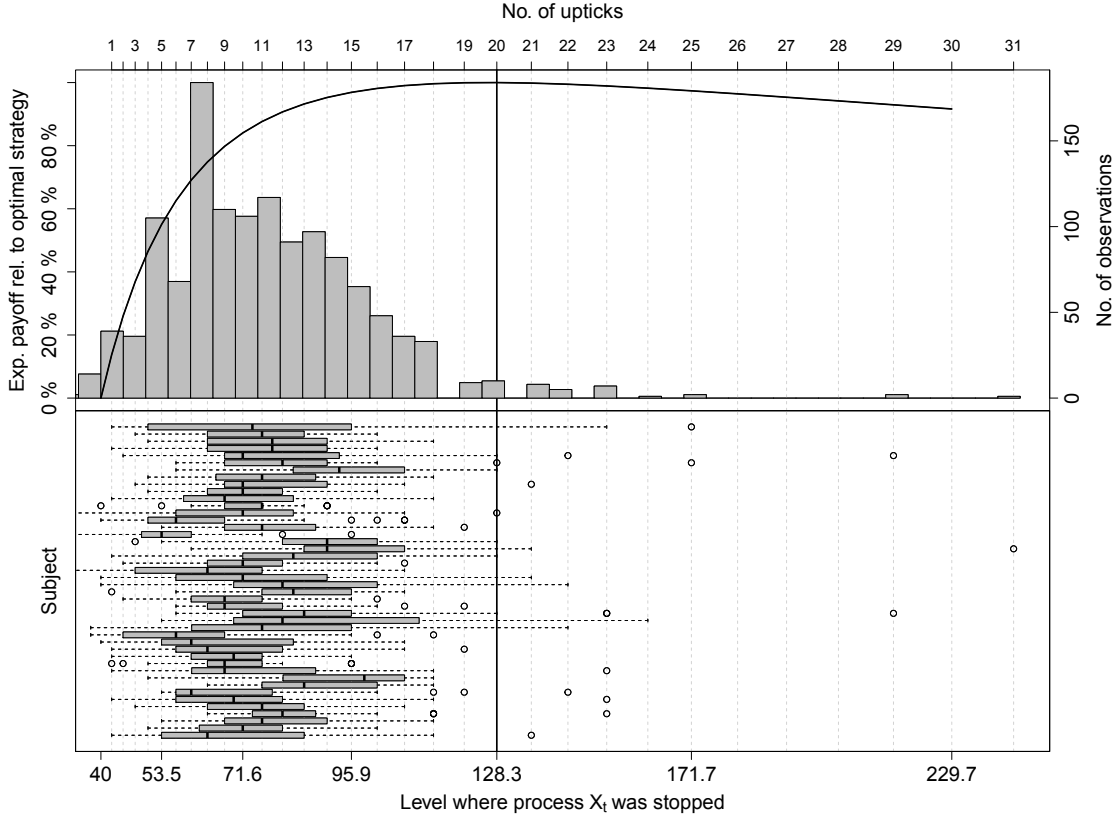


Figure 2.2.: Line in the upper chart shows how the expected payoff from using different cut-offs changes as a percentage of the payoff under the optimal strategy (left scale), grey bars show a histogram of actual reservation levels (right scale). Boxplots in bottom chart illustrate the variation in reservation levels at the subject level. The vertical line indicates the optimal cut-off for a risk neutral agent.

level across different rounds. We then inspect in how far their reservation level is constant or time-consistent within rounds.

2.4.1. Do People have a constant reservation level?

The histogram of reservation levels in the top chart in Figure 2.2 shows that there is a large variation in reservation levels across rounds, ranging from slightly above the starting value of $X_0 = 40$ to the optimal risk-neutral level 128.3. The boxplots in the bottom chart of Figure 2.2 decompose this further into between- and within-subject variation of reservation levels. The within-subject variation accounts for 33.2% of the total variation and there is no subject for which the distance between the minimal and maximal reservation level is less than 50.41. Thus, dispersion of

Table 2.2.: Average variance of reservation levels across subjects over blocks of 10 rounds.

Rounds	1-10	11-20	21-30	31-40	41-50	51-60	60-65
Variance	366.94	503.81	232.03	311.84	481.02	335.05	123.78

reservation levels is not solely due to individual differences between subjects and quite substantial in absolute terms. This shows that subjects do not play unique reservation level strategies.

It is still possible, however, that within-subject variation is due to the gradual convergence of reservation levels: subjects could successively adapt their reservation level towards a unique level after several rounds of experimentation. If that were the case, we would expect the variation in reservation levels to decrease in the number of rounds played and to settle at some constant level. Figure B.1 in the appendix shows the observed reservation levels for each subjects over all rounds played in our experiment and from Oprea et al. (2009). In both experiments, the observed variation does not decrease in the number of rounds for the vast majority of subjects. In Table 2.2 we report the average variance of reservation levels across subjects for different blocks or rounds. Variances do not decrease on average over the course of the experiment, but fluctuate unsystematically.

Finding 1. *Subjects vary their reservation levels substantially over different rounds of the same stopping task and do not appear to converge to a unique level.*

Finding that subjects' reservation levels do not settle to a constant level does not necessarily mean that their variation is entirely unsystematic. Following Oprea et al. (2009), we therefore estimated a model on the pooled data, where subjects use a cut-off strategy $\tau(b^j)$ in every round j and adapt their reservation level b^j in response to forgone earnings in the previous round. More specifically, Oprea et al. assume that the reservation level b^j in round j follows a simple linear model, which makes the difference in reservation levels between round j and $j - 1$ a linear function of previous losses

$$b^j = b^{j-1} + K \left[\delta_E \mathbf{1}_{\{\tau^{j-1} < T\}} + \delta_L \mathbf{1}_{\{\tau^{j-1} \geq T\}} \right] (S_{\tau^{j-1}}^{j-1} - b^{j-1}) . \quad (2.1)$$

The parameters δ_E and δ_L measure an individual's sensitivity to a loss that stems from stopping below S_j and from not having stopped before the deadline, re-

Table 2.3.: Estimated effects of losses on subsequent stopping choices.

Parameter	Oprea et al.	This study
$\delta_E \times 1,000$	0.5486***	1.3873**
$\delta_L \times 1,000$	-0.9185***	-1.1227

Notes: Median estimates and p -value for the Wilcoxon signed-rank test that the distribution is centered around zero: *** $p \leq 0.01$, ** $p \leq 0.05$ and * $p \leq 0.1$.

spectively.⁷ In Table 2.3 we follow Oprea et al. and report the median of the by-subject median of δ_E and δ_L . Our estimates are qualitatively similar to that of Oprea et al.. The estimate for δ_E implies that subjects increase their reservation level, if in the previous round they observed that after stopping they could have stopped at higher values. The estimate for δ_L implies that subjects reduce their reservation level, if in the previous round the process jumped to zero before they stopped and they missed the opportunity to get a positive payoff. In our sample the adjustment to the latter is not statistically significant.⁸

To inspect how much variation between rounds can be explained by this model, we took the first observed reservation level for each subject and forecasted their reservation levels for all remaining rounds. We have plotted the results in Figure B.2 in the appendix. The plot shows that the model has limited explanatory power in our sample. In the first 20 rounds, the adaption model tracks the development of reservation levels reasonably well, but it clearly overshoots thereafter. This shows that the entire variation captured in the estimated adjustment coefficients stems from behavior in the first 20 rounds.

Finding 2. *Subjects calibrate their reservation levels during the first 20 rounds in response to forgone earnings and then stop to systematically adjust and stick with this level for all later rounds.*

⁷ We use the same estimation method as Oprea et al. (2009). That is, we obtain an estimate of δ_E for each two consecutive rounds a subject stopped by setting $b^j = X_{\tau_j}^j$ and solving for δ_E in (2.1). For each block of consecutive rounds without observing a subjects stopping decision, we may use the two adjacent reservation levels to estimate δ_L from the losses suffered due to not stopping.

⁸This is actually a common finding of most of the literature on regret and counterfactual thinking, i.e. that people experience more regret over outcomes that stem from action than from equally miserable outcomes that stem from inaction (see e.g. Kahneman & Tversky, 1982; Gilovich et al., 1998; Coricelli et al., 2005; Summers & Duxbury, 2007).

Table 2.4.: Contingency table for observed decisions.

	$X_\tau = S_\tau$	$X_\tau < S_\tau$	No. of obs.
stopped first time	326 (25%)	0 (0%)	326 (25%)
not stopped first time	205 (16%)	748 (58%)	953 (75%)
No. of obs.	531 (42%)	748 (58%)	1279 (100%)

Notes: Decisions in the upper left cell are time-consistent. Other cells represent number of deviations from the time-consistency property in terms of the maximum of the process and the multiplicity of the stopping value in the history of the process. X_t denotes the value of the process and $S_t = \max_{s \leq t} X_s$. Percentage of total observations in parentheses.

2.4.2. Do People use Time-Consistent Strategies?

In the previous section we found that subjects' behavior is not constant across different rounds. But do subjects play constant reservation levels within rounds? We first measure deviations from this time-consistency property along two dimensions: (i) we count the number of ticks the stopping value is below the current running maximum and (ii) we count the deviation in terms of *multiplicity*, i.e. we count the number of times a subject had seen her stopping value before stopping eventually.

Table 2.4 shows the results in a simple contingency table. The columns of table 2.4 contain observations that correspond to stopping *at* the running maximum (left column) and below the running maximum (right column). Hence, 42% of observations stop at the maximum of the process. The remaining 58% do not. The rows of tab. 2.4 contain observations that correspond to stopping decisions that stopped *the first time* the process reached this value (top row) and decisions that stopped afterwards (bottom row). We observe only 326 out of 1279, i.e. roughly 25%, decisions that are perfect cut-off decisions. The remaining 75% are not.

Finding 3. *Subjects stopping decisions are not time-consistent 75% of the time.*

It is worth to stress that the above finding is a strong result in our setting. In fact, this renders 75% of all observed stopping decisions are irreconcilable with any of the choice models we presented earlier.

To shed more light on the magnitude of time-inconsistency, we have plotted a histogram of the multiplicity of subjects' stopping decisions in Figure 2.3. The average multiplicity of subjects' stopping decisions is roughly 3, i.e. subjects

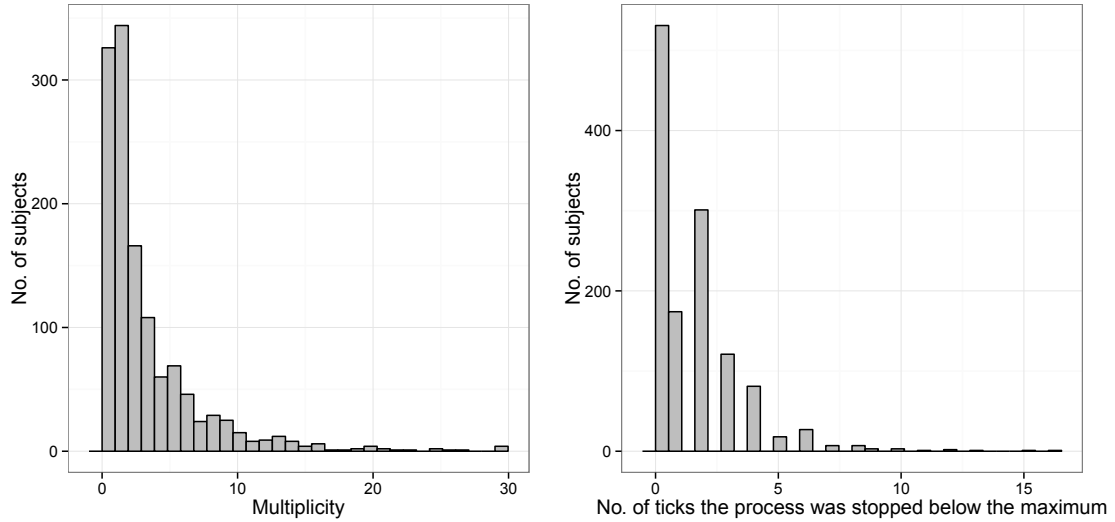


Figure 2.3.: The empirical distribution of the multiplicity of stopped values for all subjects (left) and the number of ticks subjects stopped below the previous maximum (right).

on average stop at a point they have seen three times before. The maximum multiplicity is equal to 29. About 35% of the observed stopping decisions have a multiplicity greater two.

In light of these findings, subjects clearly do not play cut-off strategies. Instead they vary their reservation levels over different rounds of the same stopping task, do not behave time-consistently 75% of the time and visit the same level of the process on average three times before they eventually stop at it.

An interesting feature of the observed behavior is that stopping seldom occurs appreciably below the past maximum. That is, while 59% of the observed stopping decisions are such that stopping occurs below the past maximum. The histogram on the right of figure 2.3 shows that the difference between the stopped value and the past maximum is seldom more than two or three ticks.

Finding 4. *Subjects show a disposition to stop near the past maximum of the process and not to stop appreciably below.*

Given that in the experiment there are two ticks every second, it seems possible on the one hand that subjects attempt to stop at the maximum the first time, but their limited reaction time often leads to miss the exact maximum and fall slightly below. On the other hand, more systematic forces such as an aversion to realize a loss relative to the past maximum could lead subjects to deliberately waive any

smaller values. In the next section, we put forward a structural econometric model that is capable of disentangling the two possible explanations in a likelihood sense.

2.4.3. Dynamic stochastic choice

The observed deviations from cut-off behavior can be explained by introducing a stochastic component alongside our structural or deterministic models of choice. Not only does such a model open up a way to accommodate salient features of our data, it also provides a way to test an expected utility model against a model with regret.

In our model the process that leads the agent to stop or continue in period t is influenced by the level of the process X_t and – in the case of regret – by the running maximum S_t . As in the deterministic case, the choice between stopping and continuation at a point (x, s) is determined by the difference between the stopping value, we denote by $sv(x, s)$, and the continuation value, we denote by $cv(x, s)$. The stopping value $sv(x, s)$ is the utility from stopping at a given point (X_t, S_t) , i.e.

$$sv(x, s) = u(x - K) - \kappa u(s - K) , \quad (2.2)$$

and $cv(x, s)$ is the expected utility from rejecting the current offer in favor of future offers. With probability $1 - \delta$ the process jumps to zero in the coming period and continuation yields a payoff of $u(0) - \kappa u(s - K)$. With probability δ the process does not jump to zero, but increases or decreases by one tick. Hence, the agent will continue from hx with probability p , and from $h^{-1}x$ with probability $(1 - p)$. Thus, using Equation (1.3)

$$cv(x, s) = \begin{cases} \delta [pV^*(xh, s) + (1 - p)V^*(xh^{-1}, s)] - (1 - \delta)\kappa u(s - K) & \text{if } x < s , \\ \delta [pV^*(xh, sh) + (1 - p)V^*(xh^{-1}, s)] - (1 - \delta)\kappa u(s - K) & \text{if } x = s . \end{cases}$$

In contrast to the deterministic case, there are now two additional factors that affect choice. First, there is an unobserved factor ϵ_t . We assume that ϵ_t is a random shock to the current utility difference between stopping and continuation. One may view ϵ_t as a literal error term or some outside unobservable force, e.g. a random taste shock, that hits the agent. We assume that ϵ_t is i.i.d. $\mathcal{N}(0, \sigma^2)$. An agent's choice in period t is then viewed conditional on the realization of ϵ_t and is

given by the choice function

$$\psi(x, s, \epsilon) = \begin{cases} \text{stop} & \text{if } \text{sv}(x, s) - \text{cv}(x, s) + \epsilon \geq 0 \\ \text{continue} & \text{if } \text{sv}(x, s) - \text{cv}(x, s) + \epsilon < 0 , \end{cases}$$

Note that in order to calculate the expected utility from continuation with the optimal strategy V^* , we need to evaluate the expected utility from continuation for the regret agent $\mathbb{E}_{t,x,s} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K)] - \kappa \mathbb{E}_{t,x,s} [u(S_{\min\{\tau, T\}} - K)]$. In particular, this requires an expression for the anticipated regret, which is given in the following lemma proven in the appendix

Lemma 8 (Anticipated Regret). *The anticipated regret $\rho(x, s, b)$ associated with the cut-off strategy $\tau(b)$ when being at point (x, s) equals*

$$\begin{aligned} \rho(x, s, b) &= \mathbb{E}_{t,x,s} [u(S_{\min\{\tau(b), T\}} - K)] \\ &= \kappa \sum_{i=0}^{m-1} \left(\frac{x}{xh^i} \right)^\alpha (1 - h^{-\alpha}) \max\{u(s - K), u(xh^i - K)\} \\ &\quad + \kappa \left(\frac{x}{b} \right)^\alpha \max\{u(s - K), u(b - K)\} . \end{aligned}$$

where $m = \frac{\log(b/x)}{\log(h)}$.

For estimation, we assume that agents' utility is of the power-utility form

$$u(x - K) = \begin{cases} \frac{K}{\theta} \left[\left(\frac{x}{K} \right)^\theta - 1 \right] & \text{for } \theta \neq 0 , \\ K \ln \left(\frac{x}{K} \right) & \text{for } \theta = 0 . \end{cases} \quad (2.3)$$

However, simultaneous estimation of the error variance σ^2 and the curvature of the utility θ generally is a delicate issue in models of stochastic discrete choice (see e.g. Wilcox, 2011, for a discussion of such issues in static models). The reason in our case is that a large variation in reservation levels across rounds leads to a relatively large error variance. A high error variance, however, translates into a low probability to reach higher levels of X_t a posteriori. In order to reconcile that we do observe subjects stopping at high and low values of X_t , the model needs to make subjects less risk averse because that increases their likelihood to reach them. Hence, there is a purely mechanical relationship between σ and θ which somewhat blurs the interpretation of θ as a measure of risk aversion in the sense of Pratt (1964).

We therefore incorporate a second terms that affects choice: inattention or attentional lapse. This comes in the form of a constant probability w . In every period t the agent does not pay attention with probability w , and continues with probability one. We assume that w is constant across all rounds. Since we do not observe w , we have to estimate it from the data. Note that under this assumption the time between two periods that the agent does pay attention is exponentially distributed and $(1 - w)$ is the rate of the distribution. Then we may interpret $(1 - w)^{-1}$ as the average time between to periods the agent pays attention. Given that in our experiment there are two ticks per second, it does not seem too far fetched to assume that most subjects will not pay close enough attention to warrant the assumption that they deliberate twice per second whether to stop or not. From a purely mechanical point of view, the probability w provides a way to incorporate variation into reservation levels without an unduly upsurge in variance and risk tolerance of an agent.⁹

For estimation, denote by $\hat{\psi}_t = 1$ and $\hat{\psi}_t = 0$ the observation that the agent decided to stop or continue in period t respectively and let $\gamma = [\theta, \kappa, \sigma, w]$ be the vector of parameters to estimate. The probability that an agent chooses to continue at a given point (x, s) then equals

$$\mathbb{P}[\text{continue at } (x, s) \mid \gamma] = (1 - w)\Phi_\sigma(\text{cv}(x, s \mid \theta, \kappa) - \text{sv}(x, s \mid \theta, \kappa)) + w$$

where Φ_σ denotes the normal cdf with mean zero and variance σ^2 . To shorten notation we will henceforth denote the conditioning on the parameters by \mathbb{P}_γ . Note that our model entails that mistakes are more likely to occur, if it is less costly for the agent, i.e. when the difference between stopping and continuation value is small.

If we let $Y^{(i,j)} = \{X_t^{(i,j)} = x, S_t^{(i,j)} = s, \hat{\psi}_t^{(i,j)}\}_{t=1}^T$ denote the available data for agent i in round j , the likelihood to observe that the agent stops in period τ

⁹We have fitted a model without an inattention parameter w and our results confirm that θ and σ both inflate drastically. The implied reservation levels lie above 99% of the observed decisions and thus the model has to attribute almost all observed stopping decisions to the rror term.

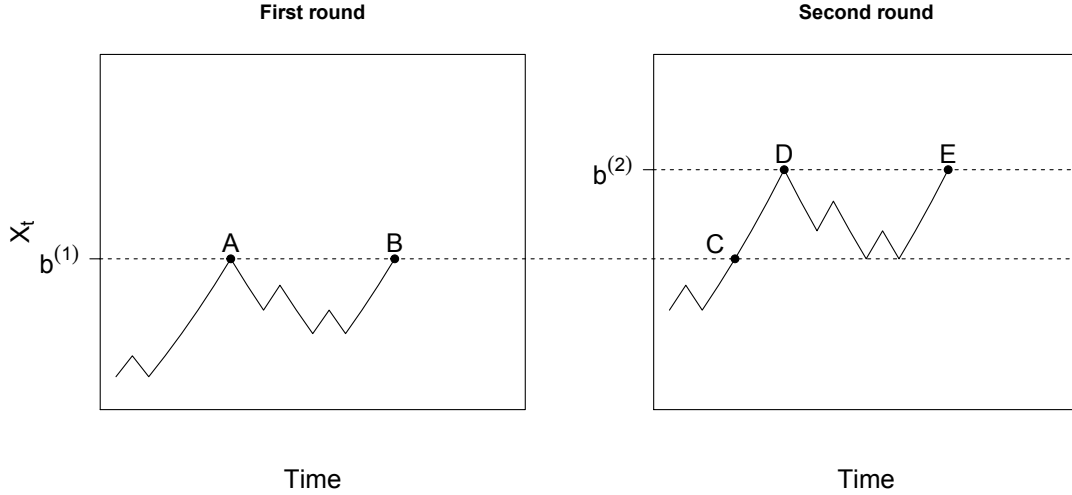


Figure 2.4.: Illustrates how a regret model can be distinguished from an EU model over rounds of the experiment.

equals¹⁰

$$\begin{aligned}
 p(Y^{(i,j)} \mid \gamma) &= \mathbf{1}_{\{\tau < T\}} (1 - \mathbb{P}_\gamma [\text{continue at } (X_\tau^{(i,j)}, S_\tau^{(i,j)})]) \\
 &\quad \times \prod_{t=1}^{\min\{\tau-1, T\}} \mathbb{P}_\gamma [\text{continue at } (X_t^{(i,j)}, S_t^{(i,j)})] \quad (2.4)
 \end{aligned}$$

As we mentioned before, the model provides us a way to judge whether the fact that subjects tend stop near the past maximum of the process is significantly related to regret or not. To see how this works, consider behavior over two consecutive rounds. Suppose the agent observes the two processes depicted in Figure 2.4 and stops at B in the first round. The agent thus missed to stop at point A, i.e. the first time the level $b^{(1)}$ was reached. Under EU, the most likely explanation for this decision in isolation is that the agent committed just a single error at point A. Hence, fix θ such that $b^{(1)}$ is the ex ante optimal cut-off $b^u = b^{(1)}$. Similarly, fix the parameters (θ, κ) for a corresponding regret agent such that $b^r = b^{(1)}$.¹¹ In the next round, suppose that the agent stopped at point E, i.e. at a higher value than in the previous round $b^{(2)} > b^{(1)}$. For given parameters, the EU model can

¹⁰We are free to assume any value for $\hat{\psi}_t$ after stopping occurred, because these periods do not enter the likelihood function and are payoff-irrelevant. Note that the way we specify the likelihood already takes into any effects of censoring and thus already corrects for any bias that stems from rounds that subjects did not stop before T .

¹¹According to Proposition 4, there always exists such a tuple.

only reconcile this with the decision in the previous round by assuming that for all points on the segment between C and E the subject erroneously continued to play. As long as $b^{(2)} \leq b^r$, the regret agent only errs at the points on the segment from C to D. For all points on the line segment from D to E, the agent does not want to stop by Lemma 6. The regret agent's continuation value is thus larger than the stopping value. In terms of the likelihood given in equation (2.4), the latter fact implies a higher second-round likelihood for the regret model. Intuitively, the regret model is able to attribute more variation in behavior to the structural part of the model and less to noise. Thus, if the described pattern is abundant in the data, the regret model finds more support in terms of any likelihood criterion. If the fact that subjects stop close to the past peak is merely because they missed to stop at exactly the maximum due to some attentional lapse, the likelihood of the regret model will only be marginally higher. Then any criterion that sufficiently penalizes the regret model for its additional degree of freedom would prefer the more parsimonious EU model. Additionally, the parameter w provides a way for the model to attribute any such unsystematic variation to inattention. Hence, any regret aversion that we find in our model, may be regarded as being net of inattention or reaction time.

We use Bayesian inference to estimate the model. Given a prior distribution $p(\gamma)$ and the joint posterior density is proportional to prior times likelihood

$$p(\gamma \mid Y) \propto p(Y \mid \gamma)p(\gamma) . \quad (2.5)$$

In the prior distribution we assume that θ , κ , w and σ are *a priori* independent

$$p(\gamma) = p(\theta)p(\kappa)p(w)p(\sigma) . \quad (2.6)$$

For κ and w we set the prior equal to the uniform distribution over the interval $[0, 1]$, for θ and σ we set an uninformative (Jeffreys) prior (Jeffreys, 1946)

$$p(\gamma) = \text{Uniform}(0, 1) \times \text{Uniform}(0, 1) \times \frac{1}{\sigma} . \quad (2.7)$$

The joint posterior density is not of any known form and thus there is no direct way to sample from it. We therefore first find the posterior modes of (2.5) with a standard hill-climbing algorithm (BFGS) and take these as starting values for a Markov Chain Monte Carlo algorithm (Metropolis-Hastings) that simulates

the joint posterior of all unknowns (Metropolis, Rosenbluth, Rosenbluth, Teller & Teller, 1953). The Metropolis-Hastings algorithm iterates over the followings steps:

1. **Step 0 (Initialization):** Choose a starting values $\gamma^{(0)}$ and a *candidate-generating* or *proposal* density $Q(\theta^{(s+1)} | \gamma^{(s)})$.
2. **Step 1 (Proposal):** For iteration s generate a proposal $\hat{\gamma}$ from Q based on $\gamma^{(s-1)}$.
3. **Step 2 (Accept/Reject):** Accept the current draw, i.e. set $\gamma^{(s)} = \hat{\gamma}$, with probability

$$\alpha = \min \left\{ 1, \frac{p(\hat{\gamma} | X)}{p(\gamma^{(s-1)} | X)} \right\} , \quad (2.8)$$

or reject the candidate with probability $1 - \alpha$, i.e. set $\gamma^{(s)} = \gamma^{(s-1)}$.

4. **Step 3 (Iterate):** Go back to Step 1.

This algorithm (i) accepts draws that are *a posteriori* more likely than the previous draw with probability $\alpha = 1$ and (ii) does not require expensive computation of the normalizing constant of the posterior density $p(\gamma | X)$ to evaluate α for each sweep of the sampler. As starting values for γ we take the posterior modes, which we determine using a standard gradient methods.¹²

Proposals are then drawn according to the recursion

$$\begin{bmatrix} \hat{\theta} \\ \hat{\kappa} \\ \hat{\sigma} \\ \hat{w} \end{bmatrix} = \begin{bmatrix} \theta^{(s-1)} \\ \kappa^{(s-1)} \\ \sigma^{(s-1)} \\ w^{(s-1)} \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \end{bmatrix} ; \quad \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \end{bmatrix} \sim \mathcal{N}(0, \Sigma) , \quad (2.9)$$

hence the name *random walk* Metropolis-Hastings. The proposal density Q is adapted while the sampler is iterating. We start the sampler by setting Σ equal to the diagonal inverse Hessian or Fisher information matrix at the posterior mode. After the first 100 iterations, the matrix Σ is adapted every 100 iterations and its

¹²We use the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm.

Table 2.5.: Summary statistics for the posterior modes.

	Exp. utility			Regret			
	θ	σ	w	θ	κ	σ	w
Min.	0.18	1.96	0.00	0.37	0.00	1.16	0.00
1st Qu.	0.70	3.90	0.75	0.78	0.00	3.33	0.76
Mean	0.85	4.91	0.74	0.94	0.10	4.32	0.75
3rd Qu.	1.03	5.70	0.92	1.11	0.18	5.35	0.94
Max.	1.29	8.63	0.97	1.46	0.36	7.40	0.99

diagonal elements are set to

$$\Sigma = \frac{0.5s - 1}{0.5s} \text{diag}(\widehat{\text{Var}}(\theta^{(0.5s:s)}), \widehat{\text{Var}}(\kappa^{(0.5s:s)}), \widehat{\text{Var}}(\sigma^{(0.5s:s)}), \widehat{\text{Var}}(w^{(0.5s:s)}))$$

where

$$\widehat{\text{Var}}(\theta^{(0.5s:s)}) = \frac{1}{0.5s - 1} \sum_{i=0.5s}^s (\theta^{(i)} - \bar{\theta})^2 \quad (2.10)$$

and $\bar{\theta}$ denotes the sample mean of $\theta^{(i)}$ for $i = 0.5s, \dots, s$.¹³ Note that adaption helps tuning the algorithm. If Σ is such that proposed jumps through the parameter space are excessively large, this will result in a high rejection rate and thus inefficient sampling. On the other hand, if Σ is such that Q only produces small jumps through the parameter space, the sampler will be slow mixing and not explore the parameters space sufficiently.

Table 2.5 and Figure 2.5 show summary statistics and histograms of the posterior modes. We find that the average subject is mildly risk-averse under the EU model and virtually risk-neutral under the regret model. This implies that with regret the observed behavior is characterized by regret aversion, whereas risk-aversion seems to play a negligible role. The probability w is found to be relatively high in our sample. In terms of attentional lapse, the numbers imply that the average subject in our sample makes decisions every 1.92 seconds in the EU model and every 2 in the regret model. This seems to be a plausible amount of inattention in our experiment, where the process evolves relatively fast.

We have plotted the simulated marginal posterior for κ per subject in Figure B.3 in the appendix. We find that for the majority of subjects, the bulk of posterior

¹³Haario, Saksman & Tamminen (2001) for details. Their algorithm is implemented in the R package `MHadaptive` (Chivers, 2012).

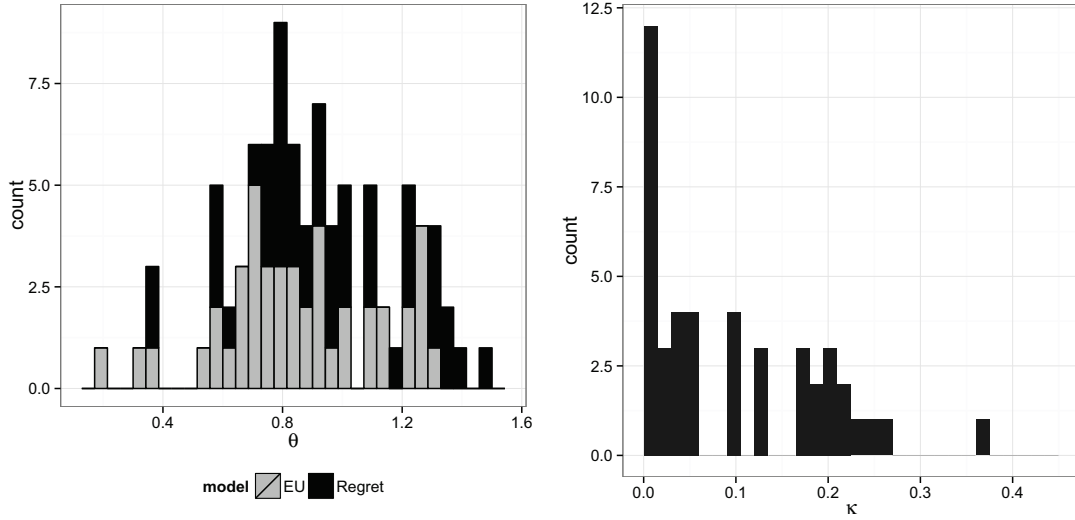


Figure 2.5.: Histogram of posterior modes of θ and κ .

mass for κ is not at or close to zero, but that there is much posterior evidence that κ is greater zero. The width of some of the posterior distributions also indicates that κ is not easily identified from the data. Does the regret model therefore fit the data better than the EU model? To have a fair comparison between the two models, we have to take into account that the regret model has one additional degree of freedom. We have therefore computed Akaike's information at the posterior mode

$$AIC = -2LLF + 2 * k ,$$

where LLF denotes the log-likelihood of the model and k stands for the number of parameters in the model. The smaller the AIC, the better the fit of a model. Because the AIC rises in the complexity of the model, i.e. in the number of parameters, more parsimonious models are preferred. In Figure 2.6 we have plotted the differences between AIC of the EU and the regret models of all subjects. The size of the points in the figure increases with the value of the posterior mode of κ . According to the AIC, the regret model is preferred for 14 of our subjects, i.e. 32%.

Finding 5. *There is significant evidence for regret aversion in our sample. One third of our subjects systematically avoid to take a loss relative to the past maximum of the process, because they are regret-averse.*

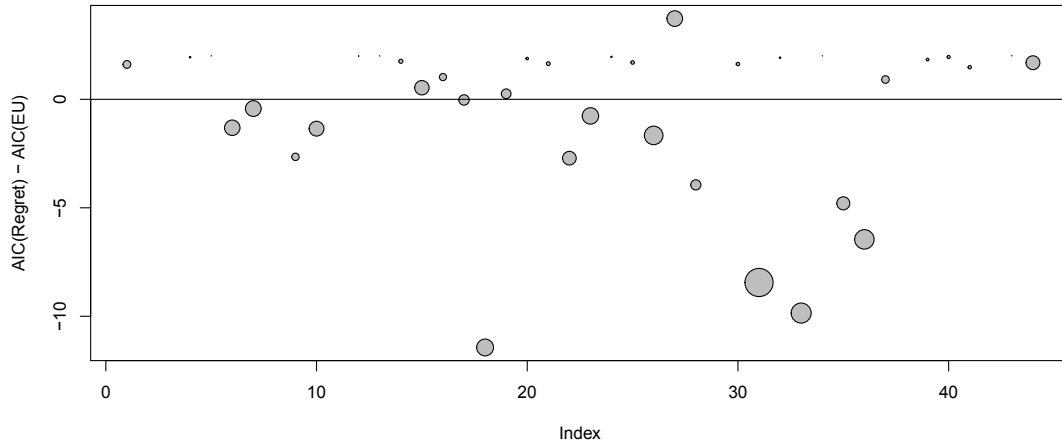


Figure 2.6.: Shows the difference between the AIC of the EU and the regret model.

2.5. Conclusion

A classical theoretical result from the optimal stopping literature is that the optimal strategy of an agent maximizing expected utility is to wait for the payoff process to reach a given reservation level and then stop immediately. In the first chapter of this dissertation, we have demonstrated that this is a very robust prediction about behavior. Moreover, our results made it very explicit that time-consistency is a primitive feature of this prediction across different theories. The existing experimental literature on stopping behavior mostly concludes that subjects in the laboratory are able to closely approximate the risk-neutral optimal reservation level (Rapoport & Tversky, 1970, 1966; Oprea et al., 2009). They are mostly silent, however, with respect to time-consistency of the observed decisions.

In this paper we sought to jointly analyze the extent to which subjects in a controlled laboratory stopping task have (i) a unique reservation at which they stop and (ii) what proportion of choices is truly time-consistent. If and to which extent the time-consistency property is satisfied by subjects in the laboratory has been largely unexplored in the extant literature. Actual behavior is found to violate this property in roughly 75% of all cases, which casts doubt on the hypothesis that EU describes actual behavior well.

Since our data shows a very strong stochastic component, we further analyze

behavior under a notion of stochastic choice. That is, we consider the possibility that agents do not implement their ex-ante plan, but deviate unexpectedly from it. We have shown in the first chapter that in this case, only a regret agent will behave differently to an EU agent. The regret agent is predicted to be reluctant to stop below the past maximum. It turns out this is a choice pattern that is prevalent in our data: Subject seldom stop below the past peak, even though they do not stop there the first time it is reached. We estimated therefore an econometric model of random regret utility to test whether it helps to explain the variation in behavior better than EU. Our results confirm that a regret model produces a better fit than a corresponding stochastic expected utility model even after penalizing for additional degrees of freedom for around one third of our subjects. This finding is interesting in view of previous evidence due to Oprea et al. (2009) on the impact of foregone earnings on reservation levels. Oprea et al. document that forgone earnings from previous rounds affect choices in later rounds, while we provide evidence that foregone earnings and the associated regret also play a role within rounds.

3 Stopping with Ambiguous Payoffs

3.1. Introduction

Timing matters. Many decisions in economics are sequential timing decisions. For example, you might have to decide whether to take a job offer or decline it in favor of future offers, whether to invest your savings in the financial markets now or later or whether to have a child at the current point in your career or later. Similarly, firms may want to engage in R&D of a new technology not before market situations are 'ripe', or consider the benefits from hiring an employee given that there is some sort of dismissal protection.

In general, in an optimal stopping problem, a decision maker faces a sequence of payoffs (x_1, x_2, \dots) , drawn from some distribution. After each realization of the payoff process, the decision maker has to decide whether to continue sampling or to stop sampling and exercise her option. The payoff from stopping may either be a function of past realizations or future realizations. Both such timing problems are optimal stopping problems, in the latter case one often speaks of an irreversible investment problem.

What is common to all classical approaches to optimal stopping, is the assumption that there is only one distribution driving the payoff process and this distribution is perfectly known to the decision maker. Going from the level of a stock-listed multinational corporation to the level of the individual, the latter part of this assumption seems more and more problematic. Arguably, most decisions taken by decision makers in the real world are such that they find it hard to assign unique probabilities to future events, i.e. they are subject to ambiguity or Knightian uncertainty (Knight, 1921).

Ambiguity in individual decision making, as introduced by Ellsberg (1961), has been shown to matter in static one-shot choices in many experimental studies, the earliest one being Becker & Brownson (1964). There is less evidence on whether

and how ambiguity effects carry over to a dynamic setting. One reason might be that every empirical analysis with field data will be inevitably marred by many potential confounds. Experiments provide a way to safely control for such. Indeed, the results obtained in this paper indicate that there exists a similar effect of uncertainty in a dynamic or sequential setting and that this effect persists over a large number of repetitions of the same stopping task. Subjects in a treatment group, facing an ambiguous payoff process, invest, on average, later than subjects facing a risky payoff process. The experiment is designed in a way, that (i) the EU model predicts exactly the opposite and (ii) to control for the possibility that subjects when facing ambiguity still hold a unique, but more pessimistic prior belief. That is, under EU, those subjects facing an ambiguous payoff process should in principle stop no later than subjects facing a risky payoff process. To control for the potential confound of more pessimistic beliefs, the design exploits the observation that ambiguity averse agents in the sense of Gilboa & Schmeidler (1989) are predicted to be schizophrenic: irrespective of the possibility to take either side of a bet, the agent will always be pessimistic.

The empirical results can also be viewed as reassuring for theorists working with models of ambiguity aversion in dynamic settings. E.g. it provides an empirical foundation for theoretical models of investment behavior and portfolio choice where investors are ambiguity averse (see Epstein & Schneider, 2010, for a survey), but may also serve as microfoundation for macroeconomic models which model representative household or firm behavior with recursive multiple-prior preferences (Ilut & Schneider, 2010). More generally, the present results also provide experimental evidence for real options or Hartmann-Abel effects of higher uncertainty on irreversible investments as in Abel & Eberly (1996); Bloom (2009). In the present setting, however, the source of heightened uncertainty is not volatility (as in Abel & Eberly, 1996; Bloom, 2009), but an increase in the lack of knowledge about the distribution of payoffs.

The main finding of the paper also relates to previous results on stopping behavior under ambiguity, e.g. due to Asano, Okudaira & Sasaki (2011) and Della Seta, Gryglewicz & Kort (2013). However, the stopping problem analyzed here is fundamentally different from what has been considered in the literature so far. First, the final payoff in the experiment of this paper was *ex post* uncertain, i.e. it was not known at the time the process was stopped. In a setting with ambiguity it is important to distinguish this situation, from a setting where payoffs are *ex*

post certain, e.g. as in Asano et al. (2011) and Della Seta et al. (2013). In the given setting, this is not merely a semantic issue. In fact, model predictions about the direction of an ambiguity effect are exactly opposite across the two settings. In that sense, the findings in this paper complement the existing evidence on the effect of ambiguity on wait options. Second, unlike the other mentioned studies on the effect of ambiguity on stopping behavior, the experiment in this paper was designed such that the results are also robust to the potential confound of pessimism that could be induced by ambiguity. Without such an identification scheme, finding that subjects are more or less reluctant to stop does not necessarily indicate that subjects do not have unique prior beliefs. Third, Della Seta et al. (2013) investigate the effect of ambiguity about the probability that the investment opportunity disappears. That is, in their setting, the payoff process X_t grows deterministically, but jumps to zero with an unknown probability. In the setting investigated here, there is ambiguity about the drift of the process X_t whereas the probability that the process jumps to zero is known.

The paper is organized as follows. Section 3.2 briefly reviews the related literature. Section 3.3 motivates the underlying theoretical model by a simple example. Section 3.4 provides the general model setup and provides the central proposition to be tested. Section 3.5 describes the experimental setup and implementation in the laboratory. Section 3.6 presents the central results obtained from the statistical analysis of the data. Section 3.7 finally concludes.

3.2. Related literature

A great number of theoretical models have been developed to rationalize Ellsberg-type behavior observed in the laboratory (see *inter alia* Camerer & Weber, 1992; Mukerji & Tallon, 2004; Gilboa, Postlewaite & Schmeidler, 2008; Etner, Jeleva & Tallon, 2009; Guidolin & Rinaldi, 2010; Epstein & Schneider, 2010, for extensive reviews). Many of the existing models were subsequently taken to a dynamic setting (see e.g. Epstein & Schneider, 2010, for a review).

Theoretical foundations for optimal stopping theory under ambiguous payoff processes are mainly due to Epstein & Schneider (2003); Nishimura & Ozaki (2007); Riedel (2009); Cheng & Riedel (2013).¹ These authors take the Multiple-

¹Earlier foundations are to be found in the mathematical literature on coherent risk measures (see e.g. Artzner, Delbaen, Eber & Heath, 1999; Riedel, 2004; Föllmer & Schied, 2004) or

prior Expected Utility (MEU) model of Gilboa & Schmeidler (1989) as the basic building block and suitably adapt it to a dynamic context. Despite the widespread application of optimal stopping models in economic theory, there are relatively few papers that analyze their descriptive accuracy under risk or ambiguity.

The largest body of literature on optimal stopping tasks in the laboratory comes from the field of sequential search. For example, Amnon Rapoport and co-authors in a series of papers, investigated the individual performance of subjects in a sequential search task experimentally (Rapoport & Tversky, 1966; Kahan et al., 1967; Rapoport & Tversky, 1970; Seale & Rapoport, 1997). Similarly, there is a branch of experimental literature putting a focus on testing job search models in the laboratory, e.g. Schotter & Braunstein (1981); Cox & Oaxaca (1989, 2000). More recently, Oprea et al. (2009) adapt an experimental design replicating the theoretical environment motivated by Dixit & Pindyck (1994a). In their paper, subjects face a risky random walk and have the option to earn its current value or forego it in favor of future values. They find that subjects approximate the risk-neutral optimal strategy surprisingly well.

A first paper making a step towards checking the effect of uncertainty on decisions in the laboratory is given by Cox & Oaxaca (2000). In their experiment, however, participants are endowed with an objective prior over states of the world. Asano et al. (2011) investigate the descriptive power of the job search model by Nishimura & Ozaki (2004) when subjects have no prior information about the distribution of states of the world. Asano et al. (2011) find that observed choices support Nishimura & Ozaki's model in the sense that subjects are willing to accept lower wage offers under ambiguity than under risk. Similarly, Della Seta et al. (2013) analyze a related situation where subjects do not face a random but deterministic payoff process and have the option to earn its current value. This option may cease before execution, however, leaving subjects with zero payoff. The probability that the investment opportunity expires is ambiguous in their setting. They find that subjects react to ambiguity by exercising the option later relative to a control group, indicating ambiguity-seeking behavior.

robust control theory in macroeconomics (Hansen & Sargent, 2001).

3.3. Motivation through a simple example

This section illustrates the behavioral intuition behind the impact of uncertainty and uncertainty aversion on the decision to invest. Toward that end, I shall take the simple two-period two-state example from Nishimura & Ozaki (2007).

To ease exposition, assume a risk-neutral agent, facing an irreversible investment opportunity. For simplicity, let the one-period discount rate δ be equal zero. Time is discrete and indexed by $t = 0, 1$. The decision maker contemplates whether to invest in period $t = 0$ or in period $t = 1$ or not at all. In order to seize the investment opportunity, investment costs K have to be incurred. The immediate period-0 profit from investing is $X_0 = x_0 < K$, which is known with certainty. The period-1 payoff from the investment X_1 , however, is uncertain. It either equals x_L or x_H , where $x_H > K > x_L$ and the state H occurs with probability p_H .

Since the planning horizon is finite, we may derive the optimal strategy of the investor using backward induction. In period $t = 1$, if state H occurs, the decision maker will choose to invest, because the resulting profit is positive $x_H - K > 0$. If state L occurs, the decision maker chooses *not* to invest, because then profits are negative $x_L - K < 0$. The optimal strategy in period $t = 1$ is therefore: invest if the state is H , do not invest otherwise. In period $t = 0$, the decision maker weighs the *expected* payoff from investing in $t = 0$ against the *expected* profit from waiting until $t = 1$ and behaving optimally from there on. She postpones investment, iff waiting is profitable in expectation, i.e. if

$$p_H(x_H - K) - [x_0 - K + (p_H x_H + (1 - p_H)x_L)] > 0 \quad (3.1)$$

$$\Leftrightarrow (x_0 - K) + x_L + p_H(K - x_L) < 0 . \quad (3.2)$$

The first term in the first line is the continuation value of the option to invest, anticipating optimal behavior in $t = 1$. The second term is the stopping value of the option to invest. By collecting terms, the inequality in the second line illustrates that the lower the probability for the good state H , the more likely a decision maker is to postpone investment. Note that this effect is not obvious *a priori*, since a reduction in p_H affects both, the stopping and the continuation value, in the same direction.

In a more realistic setting, it seems natural to assume that p_H is not perfectly known to the decision maker. Suppose the decision maker has two possible *theories*

in mind, represented by probabilities $p_H \in \{.3, .7\}$. She has no or very little objective indication which theory actually prevails. Then if she adopts the theory that corresponds to $p_H = 0.3$, investment in $t = 0$ is less likely than with $p_H = 0.7$. Pessimists are reluctant to invest.

Ambiguity aversion as modeled by Gilboa & Schmeidler (1989), postulates that the decision maker will have such a set of prior beliefs or theories and evaluate expectations using the measure that is least favorable in terms of expected payoff. Since this is behaviorally equivalent to a decision maker who holds a unique, but pessimistic belief in the presence of ambiguity, an identification scheme is needed. To illustrate the identification scheme put forward here, suppose there are two possible states of the world, $S = \{R, B\}$, and the realization is not observable to the decision maker. Moreover, suppose the decision maker is not informed about the objective probability for either R or B . Before the investment decision is to be made, the decision maker has to bet on either R or B . If the decision maker predicts the state of the world correctly, the probability for x_H will be equal to $p_H > 0.5$ and $1 - p_H < 0.5$ otherwise. Under EU, preferences will be based on beliefs, and subjects will choose the state which they perceive *a priori* more likely. Hence, this puts a lower bound of 0.5 on the decision maker's prior beliefs to have chosen the true state. Compared with a decision maker that is informed that $p(R) = p(B) = 0.5$, the uninformed decision maker is thus supposed to be at least as optimistic. And based on the simple example above, they are also expected to stop no later than any subject from the informed group.

To bring this identification to the laboratory, subjects are randomly assigned to a control group endowed with an objective prior probability for R equal to 0.5 or a treatment group without objective prior. Before making the investment decision, subjects are prompted to bet on either R or B . The subsequent evolution of X_t depends on this bet as described above. Under EU, subjects in the treatment group are predicted to invest no later than subjects in the control group. The next section discusses this simple intuition in a more general setting.

3.4. The setting

Assume time is discrete, $t = 0, \dots$ and that the agent observes a sequence $\{X_0 = x_0, X_1 = x_1, \dots\}$ of realizations of a multiplicative binomial random walk. This means that for a

given starting value $X_0 = x_0 > 0$, future values are drawn recursively as

$$X_{t+1} = \begin{cases} h X_t & \text{with probability } p_H \\ \frac{1}{h} X_t & \text{with probability } 1 - p_H \end{cases}. \quad (3.3)$$

where in the following $h > 1$ will be called the step size and $p_H \in [0, 1]$ the uptick probability. Let $\mathcal{X} = \{h^q X_0 : q \in \mathbb{Z}\}$ be the set of possible states of the process X_t . Due to the one-to-one relation between upticks q and the level of the process x , I may represent the set \mathcal{X} also by the integer grid \mathbb{Z} . Note that since the process is fully resorbing, the state of the process at time t is fully described by the tuple $(X_0, Q_t = q)$, where Q_t is the net number of upticks, i.e. upticks minus downticks, until period t . The uptick probability p_H is determined by whether the subject guesses the state of the world correctly. Before observing the first realization of the process, the agent has to bet on either state of the world $s \in \{R, B\}$. Let the agent's chosen state be denoted by $\iota \in \{R, B\}$. If $\iota = s$, the uptick probability $p_H > 0.5$ and otherwise $1 - p_H < 0.5$. At the end of any period there is a fixed exogenous probability $(1 - \delta) \in [0, 1]$ that the game ends and the agent receives a pay-off of zero. I denote by $T \geq 0$ the random time the game ends. At any time $t < T$ before the game ended the agent observes the realization of the random walk X_t and decides whether to 'continue' or to 'stop'. If she chooses 'stop' in period t the game ends and she receives the future values of the random walk X_t minus a constant transaction cost $K \geq X_0$ such that her pay-off equals

$$-K + \sum_{b=t}^T X_b.$$

If the agent chooses 'continue', the game ends with probability $1 - \delta$ and a pay-off of zero. If the game does not end in period t , period $t + 1$ starts and the agent observes the next realization of the random walk X_{t+1} .

3.4.1. The optimization problem of an SEU agent

An expected utility agent's objective is to maximize

$$V(\tau, Q_0 = 0) = \left[\mathbf{1}_{\{\tau < T\}} (-u(K) + \sum_{s=\tau}^{\infty} \mathbf{1}_{\{s < T\}} u(X_s h^{Q_s})) + \mathbf{1}_{\{\tau \geq T\}} u(0) \mid Q_0 = 0 \right] \quad (3.4)$$

with respect to the stopping time τ . Here u denotes the strictly increasing and concave utility function $u: [0, \infty) \rightarrow \mathbb{R}$ and $\mathbf{1}_{\{s < T\}}$ stands for the indicator function that takes the value one as long as $s < T$ and zero otherwise. Without loss of generality, let $u(0) = 0$.

Denote by $V^*: \mathbb{Z} \rightarrow \mathbb{R}$ the expected utility from stopping when the agent uses the optimal stopping strategy

$$V^*(q) = \sup_{\tau} V(\tau, q) .$$

The expectation in Equation (3.4) is evaluated using the one-step ahead posterior belief $p_{t+1|t}$ of the agent after seeing $Q_t = q$ net upticks, hence the notation \mathbb{E}_q . The agent is assumed to learn from observing X_t in a Bayesian sense, i.e. one-step ahead posterior beliefs $p_{t+1|t}(q)$ are formed as

$$p_{t+1|t}(q) = \mu(Q_t = q \mid \mu_0) p_H + (1 - \mu(Q_t = q \mid \mu_0))(1 - p_H) \quad (3.5)$$

$$\mu(Q_t = q \mid \mu_0) = \frac{A}{1 + A} ; \quad A = \frac{\mu_0}{1 - \mu_0} \left(\frac{p_H}{1 - p_H} \right)^q , \quad (3.6)$$

given a prior $\mathbb{P}(\iota = s) = \mu_0$. That is, $p_{t+1|t}$ denotes the agent's subjective belief that an uptick will occur in the coming period $t + 1$, after having seen q upticks until period t and having prior beliefs μ_0 . For notational convenience, I will omit the subscript $t + 1 \mid t$ below.

Assumption 4 (Power utility). *I assume that the agent is not risk-seeking, that u is strictly increasing in x and of the power-utility type, i.e.*

$$u(x) = x^\theta$$

with $0 < \theta \leq 1$.

The agent prefers stopping rule τ over τ' , if $V(\tau, X_0 = x_0) \geq V(\tau', X_0 = x_0)$.

As is demonstrated below, there exists a unique, optimal cut-off strategy for the EU agent.

In order to find the optimal strategy, the following lemma proves to be helpful

Proposition 5 (Stopping Value). *The expected utility from stopping after a given number of upticks $Q_t = q$, equals*

$$\Omega(q) = \frac{x_0^\theta h^{\theta q}}{1 - B(q)} - K^\theta, \quad (3.7)$$

where $B(q) = \delta [p(q)h^\theta + (1 - p(q))h^{-\theta}]$.

Proof. See Appendix. □

Armed with a closed-form expression for the stopping value Ω , the optimal stopping rule that maximizes the objective function (3.4) is found by means of recursive dynamic programming. Specifically, consider the Bellman equation of the stopping problem, which equals

$$V^*(Q_t = q) = \max \{ \Omega(q), \delta \mathbb{E} [V^*(Q_{t+1}) \mid Q_t = q] \} \quad (3.8)$$

for every $q \in \mathbb{Z}$. For a given function V^* , the optimal stopping rule prescribes to stop once $V^*(\tau, q) = \Omega(q)$ and to continue as long as $V^*(\tau, q) = \delta \mathbb{E} [V^*(Q_{t+1}) \mid Q_t = q]$. For notational convenience denote by A the set of values x for which stopping is optimal, i.e. the set where

$$\Omega(q) \geq \delta \mathbb{E} [V^*(Q_{t+1}) \mid Q_t = q] .$$

Because V^* is unknown, it is approximated using value function iteration. Toward that end, equation (3.8) is viewed as a functional equation. The right-hand side defines the operator $\Psi: \mathcal{V} \rightarrow \mathcal{V}$, where \mathcal{V} is the space of bounded functions and (\mathcal{V}, d_∞) denotes the associated complete metric space equipped with the supremum or uniform metric $d_\infty(V, W) = \sup_{q \in \mathbb{Z}} d(V(q), W(q))$, where d is a metric, e.g. $d_1(x, y) = |x - y|$, on \mathbb{R} .

If I let $W \leq V^*$ stand for $W(q) \leq V^*(q)$ for all $q \in \mathbb{Z}$. It then holds for Ψ that

$$\begin{aligned} \Psi(V^*) &= \mathbf{1}_{\{A\}} \Omega(q) + (1 - \mathbf{1}_{\{A\}}) \delta \mathbb{E} [V^*(Q_{t+1}) \mid Q_t = q] \\ &\geq \mathbf{1}_{\{A\}} \Omega(q) + (1 - \mathbf{1}_{\{A\}}) \delta \mathbb{E} [W(Q_{t+1}) \mid Q_t = q] = \Psi(W) . \end{aligned} \quad (3.9)$$

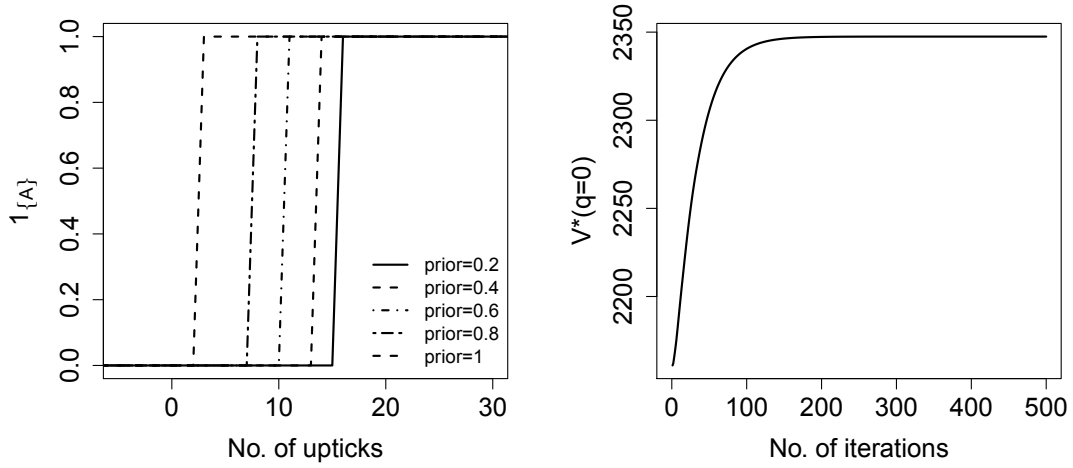


Figure 3.1.: Stopping regions (value = 1) for an agent with $\theta = .85$ and several priors (left panel) and the behavior of the value function at the origin ($q = 0$, $\mu_0 = 0.4$) over 500 iterations on the value function (right panel).

Also note that for any $k > 0$

$$\Psi(V^* + k) = \Psi(\Omega(q) + \delta \mathbb{E}[V^*(Q_{t+1}) + k \mid Q_t = q]) = \Psi(V^*) + \delta k . \quad (3.10)$$

Equations (3.9) and (3.10) together imply that Ψ satisfies Blackwell's sufficient conditions (Stokey, Lucas & Prescott, 1989). Hence, the mapping Ψ is a contraction with modulus $\delta < 1$, i.e. it holds that for any $V_0 \in \mathcal{V}$,

$$d(\Psi^n(V_0), V^*) \leq \delta^n d(V_0, V^*) .$$

Consequently, by iterating on the mapping Ψ from any starting point $V_0 \in \mathcal{V}$, one will converge to the fixed point V^* , i.e. to the value function underlying (3.8). Furthermore, the fact that Ψ is a contraction also ensure that the fixed point V is unique (see Stokey et al., 1989, for a proof).

Since V is a fixed point of Ψ , it may be found recursively by starting with an initial guess $V_0 \in \mathcal{V}$ and determining every following V_i as

$$V_{i+1} = \Psi(V_i) .$$

By the Contraction Mapping Theorem, the sequence $\{V_i\}$ converges uniformly to V^* .

Figure 3.1 shows the results from iterating on the value function for a given parameter constellation until convergence. The left-hand panel depicts the number of upticks from the starting value against the indicator function $\mathbf{1}_{\{A\}}$, where A again denotes the stopping region. Hence, the stopping region is found to be a connected set and the boundary of this set moves to the left for increasing prior belief μ_0 . Hence, initially more optimistic agents are predicted to stop earlier.

Note that we may find the optimal strategy of an agent with multiple priors in the same way. An MEU agent possesses a *set* of prior beliefs \mathcal{M}_0 on s and following Epstein & Schneider (2003, 2007), this set of prior beliefs is updated prior-by-prior according to Bayes rule

$$\mathcal{P}_{t+1|t}(q) = \{\mu(Q_t = q \mid \mu_0) \cdot p_H + (1 - \mu(Q_t = q \mid \mu_0)) \cdot (1 - p_H) : \mu_0 \in \mathcal{M}_0\} . \quad (3.11)$$

Given this sequence of beliefs, the decision maker evaluates future prospects with respect to all measures in $\mathcal{P}_{t+1|t}$ and then takes the measure that yields minimal expected utility. In the given setting, the worst case at each point q is always a future downtick, i.e. the expected payoff of the agent is a monotone function of the net number of upticks q . To see this, note that posterior beliefs increase in the number of upticks $\frac{\partial}{\partial q}p(q) > 0$ and thus

$$\frac{\partial}{\partial q}B(q) = \frac{\partial}{\partial q}\delta [p(q)h^\theta + (1 - p(q))h^{-\theta}] > 0$$

because $h^{-\theta} < h^\theta$ under the given assumptions. Consequently, the denominator in (3.7) is decreasing in q . Since $h > 1$ the numerator is increasing in q and therefore $q \mapsto \Omega(q)$ is increasing. It then also follows that the expected utility $V(\tau, Q_t = q)$ from using the stopping strategy τ is increasing in q . Therefore the worst-case measure $\underline{p}(q)$ in the set $\mathcal{P}_{t+1|t}(q)$ is always the one associated with the prior $\underline{\mu}_0 \in \mathcal{M}_0$ that assigns least prior probability to an uptick (see Riedel, 2009, section 4.2 for a discussion).

It then follows that the optimal strategy for an agent with multiple priors can be found in the same fashion as under expected utility, where now all expectations are evaluated using the worst-case one-step ahead probabilities. Hence, for the MEU agent it is optimal to stop at the highest q^* that is in the set of cut-offs associated with \mathcal{M}_0 .

3.5. Implementation

The experiment was conducted as a computer-based experiment. The experimental software was programmed using Java and Python and ran in a browser. Subjects saw two different screens in each of the 45 rounds that they played. Subjects were told that they have the option, but not the obligation, to invest in a factory, which, upon investment, produces one unit of a fictitious product every period until the end of the round.

In the laboratory, the experiment was implemented by a series of two alternating screens (see fig. C.3 and C.4). The first screen prompted subjects to set a color for the coming round. The screen showed a simple radio button for each color R and B and subjects had to click a button below to confirm their choice (see fig. C.3). Subjects were told that the behavior of the process X_t would depend on whether they met consumers' taste for the color of the product. There were two colors available, red (R) and black (B). It was then mentioned that prior to each round, they would have to set up their machines such that they produce red or black products, but not both, for the entire round. If the color they chose matched consumers' taste, the per-period profits had a 57% chance experiencing an uptick. If the chosen color did not match consumers taste, the probability for an uptick was $1 - 0.57 = 0.43$ or 43%. The choice for the state-wise probability for an uptick versus a downtick was largely dictated in order to meet a sweet spot between the amount of uncertainty and informativeness of the realizations. First, the greater the difference between p_H and $1 - p_H$, the greater the difference between the respective drifts of the per-period profit processes. For a negligible difference, the impact of uncertainty is supposed to be negligible as well. Hence, from an experimental point of view, a larger difference seems desirable in order to identify an effect. Second, for any value of p_H that differs appreciably from 0.5 (by more than 0.05), the drift dominates the process visually. The true state of the world is then easily discovered after only a few observations. Note that learning decreases the amount of ambiguity over time and in the limit ambiguity disappears (Marinacci, 2002, provides a formal argument). Hence, only a small difference is pertinent to maintain ambiguity for a minimum amount of time.

The second screen presented the actual investment screen. Subjects there saw the realization of a binomial random walk with parameters mentioned above. The realizations of the payoff process X_t (measured in ECU) they observed were the

(potential) per-period profits from selling the product. This process was always started at the value of 40 ECU ($x_0 = 40$) and the factory investment cost was fixed at 3,200 ECU. Each second consisted of two ticks (see fig. C.4).

In the control group, subjects were informed that the probability for each color to be the correct one was 0.5. In the treatment group, instructions were the same as in the control group, apart from the information concerning the prior probability for either state of the world. Subjects in the treatment group, were told that the probability for red to be consumers' taste was equal to the average relative amount of rainy days per year in Jakarta (Capital of the Republic of Indonesia).² Hence, the probability for a state was linked to a real-world phenomenon, which subjects might perceive as something that could, in principle, be determined with some precision. This is in contrast to the original Ellsberg experiment (and variants thereof), where there is truly no way subjects could know or find out about the probability for any composition of the ambiguous urn.

A round ended randomly, with a given and constant probability of 0.7% (i.e. on average $t = 1/0.007 \approx 143$ ticks).³

The 45 random walks used in the experiment were the same for every subject. Based on an individual login (printed on the instructions), the set of 45 random walks was stratified over 45 rounds.⁴ The experiment was designed so that for each subject in the treatment group, there was one subject in the control group that saw the same sequence over 45 rounds (contingent on choosing the same color in a given round). This way, a potential 'round effect' is supposed to be mitigated. Otherwise subjects might be framed by particularly short/long realizations of the process in the first few rounds.

The experiment was conducted at the laboratory of the Technical University Berlin (TU) and the WZB Berlin. The dataset presented here, was obtained from three laboratory sessions with randomly selected students from the ORSEE pool of the TU and WZB.⁵ For each session 22 students from various fields of study participated in 45 rounds of the experiment for pay. Participants were randomly

²The avg. number of rainy days is roughly equal to 187 days per year, i.e. roughly 50%. See <http://worldweather.wmo.int/043/c00310.htm> (Retrieved August 22, 2013).

³A translated version of the instructions is given in the appendix to this paper.

⁴The login name consists of an initial, "A" or "B", and a two-digit number. The letter prefix indicates which treatment a subject belongs to, while the two digit number was used as the seed for a pseudo-random number generator that drew the sequence of series shown. In both groups, subjects were then numbered in an increasing order.

⁵See: <https://experimente.wzb.eu/>

assigned to either the treatment or the control group. Consequently, each group currently has 33 subjects with 45 observations each. The average duration of the experiment was 74 minutes, and the mean earnings for subjects was 15.98 Euros (median=14 Euros, minimum=5.00 Euros, maximum = 39 Euros).

3.6. Experimental results

This section outlines the results obtained from three laboratory sessions. The analysis is conducted in view of the central hypothesis derived in terms of the effect of ambiguity on subjects' behavior in the investment task. The main finding is that ambiguity leads subjects in the treatment group to seize the investment opportunity later than subjects in the control group. This effect is robust under the careful treatment of two key data properties: censoring and unobserved heterogeneity across subjects.

3.6.1. The effect of ambiguity

Due to the termination hazard, 37% of the observations in the sample are right-censored. In these cases, the process jumped to zero before a subject decided to seize the investment. Ignoring this effect in the data leads to a censoring bias in standard estimators, while dropping censored observations results in a truncated sample and leads to a truncation bias. In order to handle censoring appropriately, standard statistical inference has to be adjusted with respect to this.

Nonparametric analysis

Following Oprea et al. (2009); Della Seta et al. (2013), results are first analyzed by group using a non-parametric Kaplan-Meier or Product-Limit estimator (Kaplan & Meier, 1958). This estimator focuses on the distribution of the reservation profit. It estimates the survival function, which is the probability *not* to invest at a given value of the profit process. If N_x is the number of subjects who did not invest at a value of x *excluding* those for which the process terminated at that value, and Y_x is the number of subjects who invest at a given value of x , then the Kaplan-Meier estimator of the Survival Function is defined as

$$\hat{S}(x) = \prod_{x=0}^{x_{max}} \left(1 - \frac{Y(x)}{N(x)} \right) . \quad (3.12)$$

Table 3.1.: Mean and median reservation profit by group.

Group	No. of events	Mean ^a	SE	Median	95% CI ^b
Risk group	1017.00	64.28	1.02	41.20	[41.2, 42.44]
Ambiguity group	859.00	72.57	1.10	45.02	[45.02, 46.37]

^a Sample size N=2,970 and upper limit where integral under the survival curve is cut off equals 122.99.

^b Confidence interval for the median reservation profit.

The idea behind the Kaplan-Meier estimate is to provide a standard empirical distribution function of reservation profits x^* , taking into account that at various instances, subjects drop out of the set of subjects that still have the opportunity to invest. In the absence of censoring, (3.12) coincides with the empirical distribution function of x .

On the one hand, the Kaplan-Meier estimate provides a way to determine the direction of the effect of ambiguity, but is less suitable for gauging the magnitude of the effect. On the other hand, this procedure is truly non-parametric, hence quite robust against misspecification (e.g. see Therneau & Grambsch, 2000, chapter 2 for an in-depth discussion).

Figure C.1 shows the estimated survival function by group. As shown, the survival functions for both groups separate in a direction that contradicts the SEU model prediction. Instead, subjects in the treatment group tend to react to uncertainty in a way that is predicted by the MEU model. For a given value of the per-period profit process X_t , subjects in the treatment group have a lower probability to seize the investment. Note that under the hypothesis that subjects in the treatment group are SEU maximizer, we would expect the opposite.

One may perform a statistical test for equality of the two cdfs, by means of a log-rank test (Harrington & Fleming, 1982) with the Nullhypothesis

$$H_0 : S_1(x) = S_2(x) . \quad (3.13)$$

The associated statistic is $\chi^2(1)$ distributed. The value of the statistic is 45.76, with an associated p-value which is virtually zero.

The impact of uncertainty may be further quantified, by considering the average reservation profit within each group. Table 3.1 displays the estimates for the mean and the median reservation profit by group.⁶ In terms of the median

⁶The estimate for the mean duration suffers from a bias that stems from the fact that the

reservation profit, the magnitude of the difference between reservation profits is less striking and is roughly equal to a 9.3% increase in the reservation wage. The 95% confidence interval around the median shows, however, that the difference is significant.

Mixed proportional hazard models

In an attempt to gauge the size of the effect on reservation profits, survival or exponential regression models provide a more adequate tool. There are several widespread specifications, which mainly differ in the amount of parametric rigor they impose on the functional form of the *hazard function* $\lambda(x)$. The hazard function and the survival function are related by the equation

$$S(x) = \exp \left[- \int_0^x \lambda(p) dp \right] . \quad (3.14)$$

Hence, the hazard function is the instantaneous probability to invest at a given value of x . Proportional hazard models assume that the hazard function for individual i comprises the *baseline hazard function* and the *risk score*. It furthermore assumes that baseline hazard and risk score are related in a proportional way, i.e.

$$\lambda(x) = \lambda_0(x) \exp [Y_i \beta] \quad (3.15)$$

where y_i is the i -th row of the $(n \times k)$ -matrix of covariates and $\exp [Y_i \beta]$ is the risk score. The baseline hazard $\lambda_0(\pi)$ is treated non-parametrically (see Cox, 1972) and thus allowed to have any shape, e.g. to be increasing, decreasing or a mixture of both. An important issue is that in the present case the set of control variables Y_i is very sparse. It only comprises a dummy variable for being a member of the treatment group. Since no other personal characteristics are observed in the experiment, it is almost surely the case that there exists a substantial amount of unobserved heterogeneity among individuals. In survival regressions this leads to inconsistent estimates for the treatment fixed effect β in (3.15), if the heterogeneity is neglected. Consequently, the basic model (3.15) is extended by incorporating an individual-specific random effect θ , to absorb the unobserved heterogeneity into

survival function does not become zero over the feasible state space. Consequently, the integral has to be cut off at the highest censoring value. Comparing median and mean estimates, the bias seems to be very pronounced in the given case. The estimate of the median, however, remains unbiased.

Table 3.2.: Results from mixed proportional hazard model.

	coeff.	exp(coeff.)	SE	z-stat.	Pr(> z)
Ambiguity effect	-0.44	0.65	0.23	-1.93	0.05
LR test for random effects			-558.06	p: 0.00	

Notes: Sample size N=2,970. Efron approximation for ties.

the risk score

$$\lambda_i(x) = \lambda_0(x) \exp [Y_i\beta + Z_i\theta] . \quad (3.16)$$

It is assumed here that random effects are normally distributed

$$\theta \sim N(0, \sigma^2 I_n) . \quad (3.17)$$

The model (3.16) to (3.17) may be estimated using penalized regression methods (see Hastie & Tibshirani, 1990; Therneau, 2003).

Results from the Cox model with Gaussian frailties are given in table 3.2. The results may be easily interpreted in terms of the relative risk score, which is the probability for a subject to invest at a given value x , relative to a subject in the control group. Note that conditional on θ the *relative* hazard for a member of group i is given as

$$\frac{\lambda_{i=1}(x)}{\lambda_{i=0}(x)} = \frac{\lambda_0(x) \exp [Y_1\beta]}{\lambda_0(x) \exp [Y_0\beta]} = \exp [(Y_1 - Y_0)\beta] = \exp [\beta] . \quad (3.18)$$

Table 3.2 then reveals that the conditional relative risk score is 0.65, i.e. given the individual frailty term, ambiguity reduces the probability to invest at a given level by around 35%. However, the effect is borderline significant ($p = 0.05$). The results also show that the variance of the random effect is significantly larger than zero, as confirmed by an LR test with the null hypotheses that the variance of the frailty effect is zero.⁷

The overall results illustrate the importance to take into consideration the presence of individual heterogeneity that is due to unobserved factors.

⁷A Cox model without frailties was also estimated. In such a model, the treatment effect was significant at a 2.5% level and the relative risk score was 0.75. The proportionality assumption (3.15), however, was strongly rejected, as indicated by the scaled Schoenfeld residuals.

3.6.2. The effect of ambiguity over time

From a theoretical perspective, there is room for learning not only within a given round, but also between rounds. By observing the payoff process over rounds, subjects update their belief about the probability for either state of the world. Therefore subjects may also learn about the true probability for each state, even though the true state is never revealed to them.

Again, the results will be analyzed by means of non-parametric estimation of the survival function and mixed proportional hazard models. Figure C.5 shows the survival functions across groups for different blocks of rounds. Going from the top left panel to the bottom right panel, different blocks of ten rounds are analyzed in isolation. To allow subjects to accommodate to the experiment, the first five rounds were discarded and all subsequent rounds were divided into blocks of ten. The depicted survival curves in Figure C.5 show very much the same pattern as the overall survival curve across all 45 rounds displayed in Figure C.1. Going from the first to the last block, the effect of ambiguity seems very persistent. The difference in the survival curves, as measured by the 95% confidence bands, remains significant for the most parts of the relevant range. Moreover, even though the difference between the survival functions becomes insignificant for higher values of x , the direction of the effect never reverses into the direction implied by SEU.

The same conclusion derives from the mixed proportional hazard models, when estimated using the same subsamples of data. Tables C.1 to C.4 show the estimation results for the block of rounds mentioned before. Interestingly, the treatment effect of being in the ambiguity group gets stronger over the first three blocks until round 35. In the block that includes all decisions from rounds 25 to 35, the probability to invest at a given level of the profit process X_t is only 60% of that of a subject in the risk group. For the first three blocks, the treatment effect is always significant at the 5% level. For the last block of rounds, i.e. from rounds 35 to 45, the effect becomes insignificant and smaller in magnitude. This might be due to the fact that subjects the situation as less ambiguous over time, but it may also result from subjects' fatigue as the final rounds were usually taking place after around 60 minutes of play.

3.7. Conclusion

This paper considers the impact of ambiguity on irreversible investment decisions in the laboratory. The main finding of this paper is that laboratory data lends support to the prediction made by the multiple-prior expected utility model, as opposed to the prediction made by subjective expected utility. That is, despite the fact that subjects were given the choice to bet on either state of the world at the first stage of the experiment, they behave as if the chosen state had minimal chance to occur at the second stage of the experiment. A non-parametric analysis of observed reservation profits reveals that subjects in the treatment group have a 9% higher median reservation profit than subjects in the control group. A regression-based analysis across groups that controls for unobserved heterogeneity shows that subjects in the treatment group are 35% less likely than subjects from the control group to seize the investment at a given level of the payoff process. The effect is very persistent and statistically significant over more than thirty repetitions of the same stopping task.

These results indicate that uncertainty plays a role for an individual's decision to pledge a safe payoff in favor of an uncertain future payoff. Subjects in the laboratory ask for a higher reservation profit to forgo a safe endowment for an uncertain investment. This for examples may serve as empirical foundation for theoretical finance models of investment behavior and portfolio choice when investors are face ambiguity and are ambiguity averse. Models with ambiguity averse decision makers models are able to explain a range of common findings with respect to financial markets and investors' behavior, that are hard to reconcile with standard choice theory, such as non-participation or selective participation, portfolio inertia and non-continuous price moves in changes in fundamentals. Consequently, investors may avoid taking or even completely close positions as market conditions become more uncertain. Thus, market liquidity of certain asset classes may erode suddenly as a response to heightened ambiguity. The results obtained here indicate that it may happen due to heightened uncertainty without any increase in fundamental volatility or decrease in risk-appetite. This is even more striking, because the amount of ambiguity and complexity in the experiment may be considered as fairly small compared to real-world situations. Consequently, it seems possible that the estimated effect in this paper understates potential ambiguity premium in real-world investment decisions.

From a macroeconomic perspective, the experimental results also lend support to an uncertainty effects à la Bloom (2009). Such models argue that private households and firms react to heightened volatility by postponing investment, because the option value of waiting increases in volatility. Much along the lines of Ilut & Schneider (2010), the results from the laboratory suggest that the same reaction is triggered by an increase in ambiguity. Insofar, the experimental results may be viewed as providing a microfoundation for models that incorporate the effect of ambiguity on investment through modeling representative households with recursive multiple-prior preferences.

Appendices

A Appendix to Chapter 1

A.1. Mathematical Appendix

Proof of Lemma 1. Define the running maximum $S_t = \max_{s \leq t} X_s$. Note that

$$\mathbb{P}[\tau(b) < T \mid X_0 = x] = \mathbb{P}[S_T \geq b \mid X_0 = x] .$$

For all $x < b$, the probability of reaching the level b from period-0 perspective, is equal to the probability of reaching the next period, $T > 1$, times the expected probability of reaching b from period-1 perspective, i.e.

$$\begin{aligned} \mathbb{P}[S_T \geq b \mid X_0 = x] &= \mathbb{P}[T > 1] \mathbb{E}[\mathbb{P}[S_T \geq b \mid X_1, T > 1] \mid X_0 = x] \\ &= \delta(p \mathbb{P}[S_T \geq b \mid X_1 = xh, T > 1] \\ &\quad + (1-p) \mathbb{P}[S_T \geq b \mid X_1 = xh^{-1}, T > 1]) \\ &= \delta(p \mathbb{P}[S_T \geq b \mid X_0 = xh] + (1-p) \mathbb{P}[S_T \geq b \mid X_0 = xh^{-1}]) . \end{aligned}$$

To simplify notation define $\psi_b(x) = \mathbb{P}[S_T \geq b \mid X_0 = x] = \psi_b(x)$. By definition ψ_b is a solution to the difference equation

$$\psi_b(x) = \begin{cases} 1 & \text{for all } x \geq b \\ \delta(p\psi_b(xh) + (1-p)\psi_b(xh^{-1})) & \text{for all } x < b \end{cases}, \quad (\text{A.1})$$

taking values in $[0, 1]$. If we have two solutions $\psi_b, \hat{\psi}_b$ of Equation A.1 it holds that

$$\begin{aligned} |\psi_b(x) - \hat{\psi}_b(x)| &= \mathbf{1}_{\{x < b\}} \delta \left| p(\psi_b(xh) - \hat{\psi}_b(xh)) + (1-p)(\psi_b(xh^{-1}) - \hat{\psi}_b(xh^{-1})) \right| \\ &\leq \delta \sup_{z < b} |\psi_b(z) - \hat{\psi}_b(z)| . \end{aligned}$$

As $\psi_b(x), \hat{\psi}_b(x)$ lie between zero and one, the supremum of the differences $\sup_z |\psi_b(z) - \hat{\psi}_b(z)|$ exists and is bounded by one. As $\delta < 1$ it follows that

$$\sup_z |\psi_b(z) - \hat{\psi}_b(z)| \leq \delta \sup_z |\psi_b(z) - \hat{\psi}_b(z)| = 0$$

and thus Equation A.1 can have at most one solution taking values in $[0, 1]$. Guessing the solution of (A.1) to be of the form $\psi_b(x) = \mathbf{1}_{\{x < b\}}(\frac{x}{b})^\alpha + \mathbf{1}_{\{x \geq b\}}$ gives $1 = \delta(ph^\alpha + (1-p)h^{-\alpha})$. Substituting $z = h^\alpha$ yields the quadratic equation

$$0 = \delta(pz + (1-p)z^{-1}) - 1 = z^2 - \frac{z}{\delta p} + \frac{1-p}{p} \Rightarrow z = \frac{1}{2\delta p} \pm \sqrt{\frac{1}{4\delta^2 p^2} - \frac{1-p}{p}}.$$

By symmetry this equation has two solution of the form z, z^{-1} . Let $z > 1$ be the larger solution. For the smaller solution, $\frac{1}{z} < 1$, it follows that $\alpha = \frac{\log(\frac{1}{z})}{\log(h)} < 0$. Hence, the resulting function $\psi(x) = \mathbf{1}_{\{x < b\}}(\frac{x}{b})^\alpha + \mathbf{1}_{\{x \geq b\}}$ is decreasing in x and takes values outside $[0, 1]$. This leads to a contradiction and shows that $\alpha > 1$. Note that the function $z \mapsto \delta(pz + (1-p)z^{-1})$ is increasing for all $z \geq 1$. By Assumption 1, $\delta(ph + (1-p)h^{-1}) < \delta(pz + (1-p)z^{-1}) = 1$ and thus $1 \leq h < z = h^\alpha$ and $\alpha > 1$. \square

Proof of Lemma 3. By Lemma 2 the expected value of any cut-off strategy with cut-off $b = x_0 h^n$ is given by

$$V(\tau(x_0 h^n), x_0) = u(x_0 - K) + \sum_{j=1}^n h^{-(j-1)\alpha} \Gamma(x_0 h^j).$$

If u is concave Γ is monotone decreasing (as it follows by setting $\kappa = 0$ in the proof of Lemma 7). As $\Gamma(b^u) < 0$ it follows from the monotonicity of Γ that $\tau(b^u)$ is the optimal cut-off strategy. Denote by $x > b^u$ the point where $\mathcal{L}u(x - K) > 0$. Clearly, as it not optimal to stop at x if the optimal strategy is a cut-off strategy the optimal cut-off b must be greater x . As $x > b^u$ this is a contradiction. \square

Proof of Lemma 4. As shown in the proof of Proposition 1 $\mathcal{L}u(b^u - K) < 0$. Thus, if $\mathcal{L}u$ changes its sign at most once this implies Assumption 2. In the final step we show for constant absolute or relative risk-aversion $\mathcal{L}u$ changes its sign at most once.

Constant Absolute Risk Aversion: Suppose the agent accepts such a gamble at the wealth level x . Let $u(x) = -\frac{1}{\theta} \exp(-\theta x)$, i.e. assume the agent has constant

absolute risk-aversion of θ . The expected change in utility from waiting one more round at x equals

$$\begin{aligned}\mathcal{L}u(x-K) &= \delta(pu(xh-K) + (1-p)u(xh^{-1}-K)) + (1-\delta)u(0) - u(x-K) \\ &= \delta u(x-K) \left[\left(p \frac{u(xh-K)}{u(x-K)} + (1-p) \frac{u(xh^{-1}-K)}{u(x-K)} \right) + (1-\delta) \frac{u(0)}{u(x-K)} - 1 \right] \\ &= -\frac{e^{-\theta(x-K)}}{\theta} \left[\delta(p e^{-\theta x(h-1)} + (1-p) e^{-\theta x(h^{-1}-1)}) + (1-\delta) e^{\theta(x-K)} - 1 \right]\end{aligned}$$

We will show that the second part is monotone increasing in x . Taking derivatives of the term in square brackets gives

$$\delta(-\theta(h-1)p e^{-\theta x(h-1)} + \theta(1-h^{-1})(1-p) e^{\theta x(1-h^{-1})}) + (1-\delta)\theta e^{\theta(x-K)}$$

As $e^{-\theta x(h-1)} < 1$ and $e^{\theta x(1-h^{-1})}, e^{\theta(x-K)} > 1$ a lower is given by

$$\begin{aligned}&\geq \theta [\delta(-(h-1)p + (1-h^{-1})(1-p)) + (1-\delta)] \\ &= \theta [-\delta(hp + h^{-1}(1-p)) + 1] > 0,\end{aligned}$$

Where the last step follows as $hp + h^{-1}(1-p) < 1$ by Assumption 1. Consequently $\mathcal{L}u$ changes its sign at most once.

Constant Relative Risk Aversion: Let $u(x) = \frac{(x+K)^\theta - K^\theta}{\theta}$. The expected change in utility from waiting one more round at x equals

$$\begin{aligned}\mathcal{L}u(x-K) &= \frac{\delta}{\theta}(p(xh)^\theta - K^\theta + (1-p)(xh^{-1})^\theta - K^\theta) - \frac{1}{\theta}(x^\theta - K^\theta) \\ &= \frac{1}{\theta} \{ \delta [p(xh)^\theta + (1-p)(xh^{-1})^\theta] - x^\theta + (1-\delta)K^\theta \} \\ &= \frac{1}{\theta} x^\theta (\delta [ph^\theta + (1-p)h^{-\theta}] - 1) + \frac{1}{\theta} (1-\delta)K^\theta.\end{aligned}$$

As $p > 1/2$ for all $\theta \geq 0$

$$\frac{\partial}{\partial \theta} (ph^\theta + (1-p)h^{-\theta}) = p \log(h)h^\theta - (1-p) \log(h)h^{-\theta} \geq p \log(h)(h^\theta - h^{-\theta}) \geq 0.$$

Thus, $ph^\theta + (1-p)h^{-\theta} < ph^\alpha + (1-p)h^{-\alpha} = \frac{1}{\delta}$ for all $\theta < \alpha$, by definition of α . As $\frac{1}{\theta}x^\theta$ is increasing in x this completes the proof. \square

B Appendix to Chapter 2

B.1. Mathematical Appendix

Proof of Lemma 8. First, we derive the probability that the maximum of the process is at least $y \in \mathcal{X}$

$$\mathbb{P}[S_T \geq y \mid X_t = x, S_t = s] = \begin{cases} 1 & \text{if } s \geq y \\ \mathbb{P}[\tau(y) < T \mid X_t = x] & \text{if } s < y \end{cases}$$

Hence, we have that the probability that the maximum of the process is exactly $y \in \mathcal{X}$ for all $s < y$ equals

$$\begin{aligned} \mathbb{P}[S_T = y \mid X_t = x, S_t = s] &= \mathbb{P}[S_T \geq y \mid X_t = x, S_t = s] - \mathbb{P}[S_T \geq yh \mid X_t = x, S_t = s] \\ &= \left(\frac{x}{y}\right)^\alpha - \left(\frac{x}{yh}\right)^\alpha = \left(\frac{x}{y}\right)^\alpha (1 - h^{-\alpha}) \end{aligned}$$

Let $b = xh^m$. Given the regret functional derived in equation 1.6 the expected value of using the cut-off strategy $\tau(b)$ equals

$$\begin{aligned} V(\tau(b), x, s) &= \mathbb{E}[\mathbf{1}_{\{\tau(b) < T\}} u(X_{\tau(b)} - K) - \kappa u(S_{\tau(b)} - K) \mid X_t = x, S_t = s] \\ &= \mathbb{P}[\tau(b) < T \mid X_t = x] u(b - K) \\ &\quad - \kappa \sum_{i=0}^m \mathbb{P}[S_T = xh^i \mid X_t = x, S_t = s] u(xh^i - K) \\ &= \left(\frac{x}{b}\right)^\alpha u(b - K) - \kappa \sum_{i=0}^{m-1} \left(\frac{x}{xh^i}\right)^\alpha (1 - h^{-\alpha}) \max\{u(s - K), u(xh^i - K)\} \\ &\quad - \kappa \left(\frac{x}{b}\right)^\alpha \max\{u(s - K), u(b - K)\}. \end{aligned}$$

□

B.2. Figures

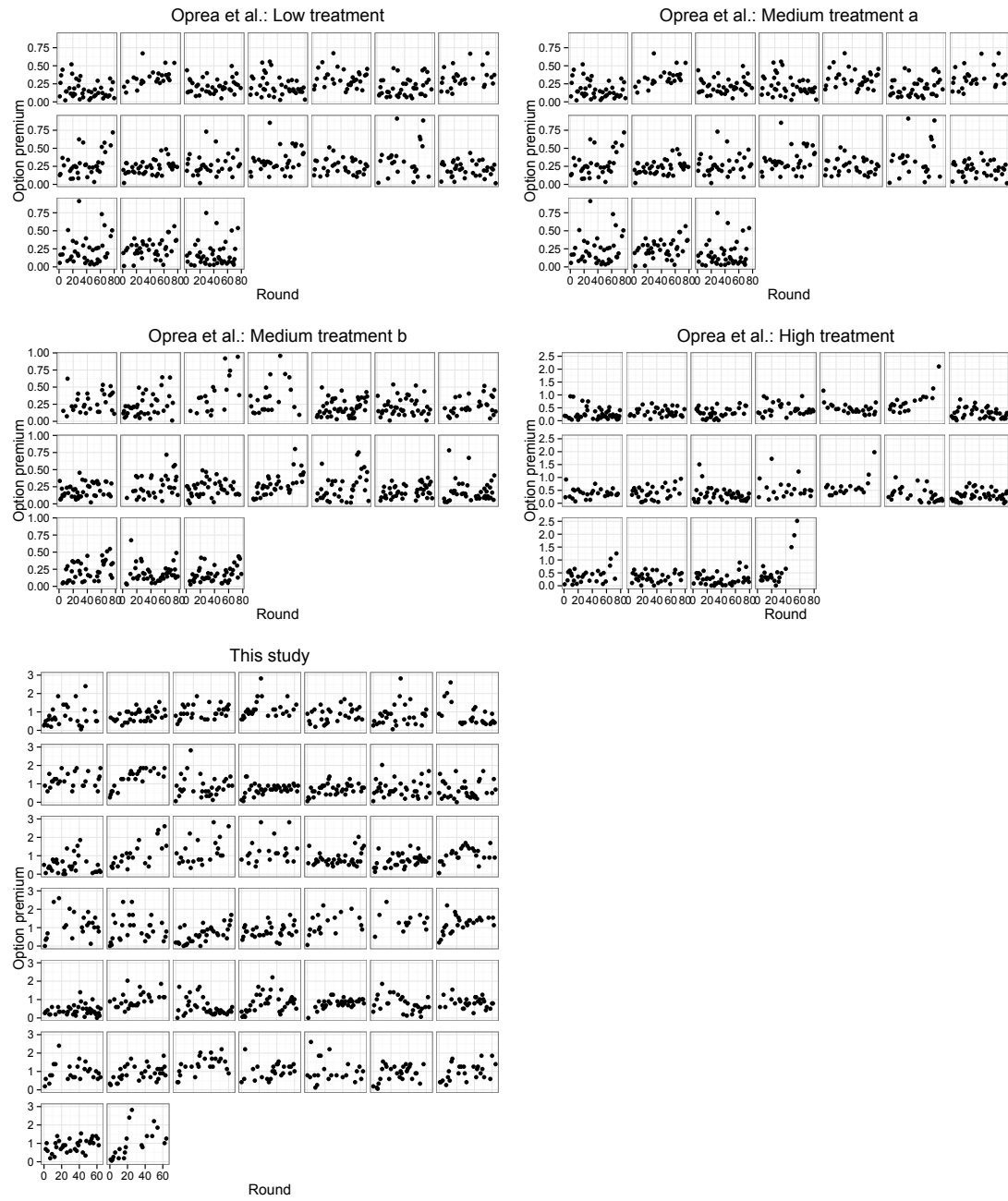


Figure B.1.: Shows the observed reservation levels for over rounds as played by subjects. Each panel in a given block of panels is the sequence of reservation levels for one subject.

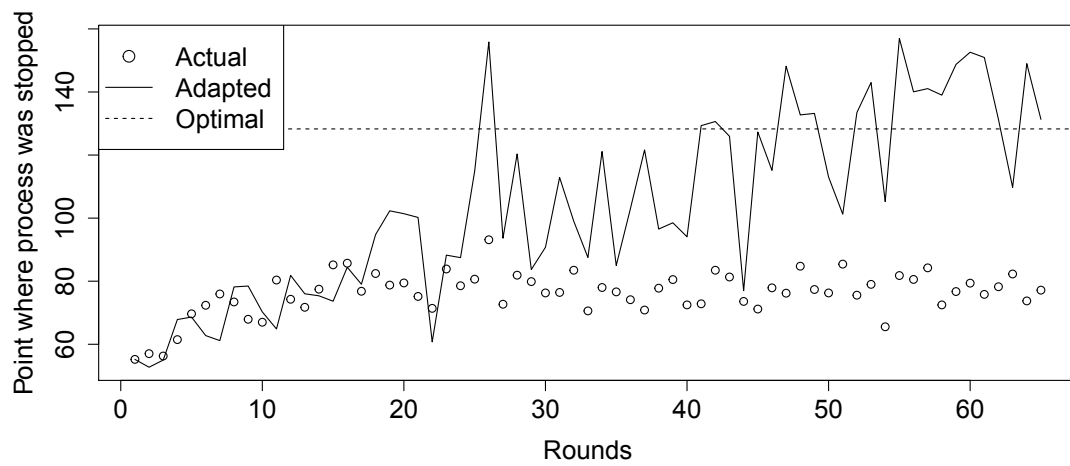


Figure B.2.: Shows simulation results from adaptive learning model versus actual choices.

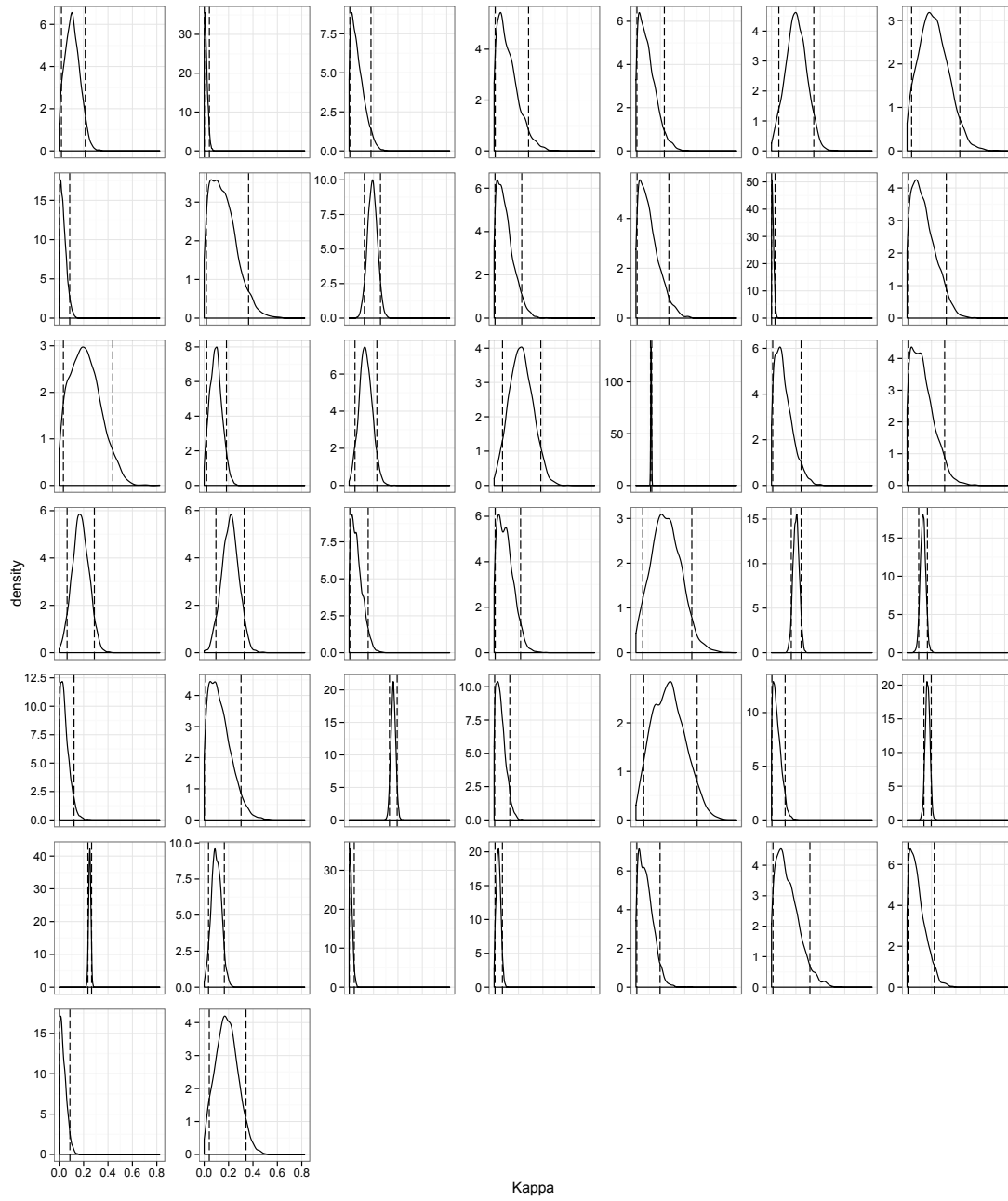


Figure B.3.: Posterior distribution of κ by subject. Dashed vertical lines depict the 95% interval.

B.3. Implementation in R

This section provides the code used for estimation of the stochastic choice model with the statistical software R (R Core Team, 2012).

B.3.1. Functions

Let us first provide some basic functions that we will use for estimating the model.

```
# Utility function
utils <- function(x, theta) {
  # utility function for CRRA  $u(x - K)$  equals:
  u <- ((x/K)^theta - 1) * (K/theta)
  return(u)
}

# Computes the Stopping value
sv.crra <- function(x, s, theta, k) {
  # computes the stopping value at a given point (x,s) for a
  # given parameter (theta,k).
  stop.val <- utils(x, theta) - k * utils(s, theta)
  return(stop.val)
}

con.val <- function(y, s, theta, kappa, steps, z, K, h) {
  theta <- theta
  k <- kappa
  xhi <- outer(array(y, length(y)), h^(0:steps), FUN = "*")
  u.xhi <- apply(xhi, 2, utils, theta)
  zs <- z^(0:steps)
  immediate <- t(apply(u.xhi, 1, function(x) x * zs))
  tmp <- immediate
  if (k > 0) {
    benchmark <- utils(s, theta)
    uvss <- t(sapply(1:length(y), function(x) pmax(benchmark[x],
      u.xhi[x, ])))
    interim <- k * (1 - z) * (t(apply(t(apply(uvss, 1, function(x) x *
      zs)), 1, cumsum)))
    reach <- k * t(apply(uvss, 1, function(x) zs * x))
    tmp <- immediate - interim - reach
  }
  res <- apply(tmp, 1, max)
  return(res)
```

```

}
cv.crra <- function(x, s, theta, k) {
  # Expected utility from waiting one more period and
  # continuing with the optimal cut-off strategy from there
  tmp <- cbind(x, s, seq_along(x))
  df1 <- unique(tmp[, 1:2])
  conval <- delta * ((p * (df1[, 1] < df1[, 2])) * con.val(h *
    df1[, 1], df1[, 2], theta, k, 120, z, K, h) + p * (df1[,
    1] == df1[, 2]) * con.val(h * df1[, 1], h * df1[, 2],
    theta, k, 120, z, K, h) + (1 - p) * con.val(df1[, 1]/h,
    df1[, 2], theta, k, 120, z, K, h)) - (1 - delta) * (k *
    utils(df1[, 2], theta))
  df2 <- cbind(df1, conval)
  ContinuationValue <- merge(tmp, df2, by = c("x", "s"))
  return(ContinuationValue[order(ContinuationValue[, 3]), 4])
}

# Next retrieve the stopping decisions from the data.
# Retrieve (i) whether or not stopping occurred and (ii) in
# which period stopping occurred.
Get.stopping.decisions <- function(Y) {
  abb <- array(, c(65, 3)) # vector of stopping decisions, 0 did not stop, 1 stopped.
  for (j in 1:65) {
    abb[j, 1] <- ifelse(Y$invested[Y$rowIndex == (j - 1)] ==
      TRUE, 1, 0)
    abb[j, 2] <- ifelse(Y$invested[Y$rowIndex == (j - 1)] ==
      TRUE, Y$investmentIdx[Y$rowIndex == (j - 1)] + 1,
      sum(!is.na(series[j, ])))
    abb[j, 3] <- ifelse(Y$invested[Y$rowIndex == (j - 1)] ==
      TRUE, data$investmentValue, NA)
  }
  return(list(abb = abb[, 1], dec = abb[, 2], lev = abb[, 3]))
}

prior <- function(Theta, prior) {
  if (length(Theta) == 3) {
    theta <- Theta[1]
    sigma <- Theta[2]
    tremble <- Theta[3]
    m.theta <- prior[1]
    s.theta <- prior[2]
    a <- dnorm(theta, mean = m.theta, sd = s.theta, log = TRUE)
    b <- dunif(tremble, min = 0, max = 1, log = TRUE)
    c <- 1/sigma
  }
}

```

```

    if (sigma == 0) {
      c <- -Inf
    }
    out <- a + b + c
  }
  if (length(Theta) == 4) {
    theta <- Theta[1]
    kappa <- Theta[2]
    sigma <- Theta[3]
    tremble <- Theta[4]
    m.theta <- prior[1]
    s.theta <- prior[2]
    a <- dnorm(theta, mean = m.theta, sd = s.theta, log = TRUE)
    b <- dunif(tremble, min = 0, max = 1, log = TRUE)
    c <- dunif(kappa, min = 0, max = 1, log = TRUE)
    d <- 1/sigma
    if (sigma == 0) {
      c <- -Inf
    }
    out <- a + b + c + d
  }
  return(out)
}

LLF <- function(Theta, data, prior) {
  Theta <- Theta * parFac
  if (length(Theta) == 4) {
    Lower <- c(1e-05, 0, 0, 0)
    Upper <- c(alpha - 0.001, 1, 1e+06, 1)
    penFac <- 1 + sum(pmax(0, Lower - Theta)^1.1) + sum(pmax(0,
      Theta - Upper)^1.1)
    Theta <- pmax(Lower, pmin(Upper, Theta))
    theta <- Theta[1]
    kappa <- Theta[2]
    sigma <- Theta[3]
    tremble <- Theta[4]
    cat("theta = ", round(c(theta), digits = 4), "kappa = ",
      c(round(kappa, digits = 4)), "sigma = ", round(sigma,
        4), "tremble = ", c(round(tremble, digits = 4)))
  }
  if (length(Theta) == 3) {
    Lower <- c(1e-05, 0, 0)
    Upper <- c(alpha - 0.001, 1e+06, 1)

```

```

penFac <- 1 + sum(pmax(0, Lower - Theta)^1.1) + sum(pmax(0,
  Theta - Upper)^1.1)
Theta <- pmax(Lower, pmin(Upper, Theta))
theta <- Theta[1]
kappa <- 0
sigma <- Theta[2]
tremble <- Theta[3]
cat("theta =", round(c(theta), digits = 4), "kappa = ",
    c(round(kappa, digits = 4)), "sigma = ", round(sigma,
    4), "tremble = ", c(round(tremble, digits = 4)))
}
tmp1 <- Get.stopping.decisions(data)
abb <- tmp1$abb
dec <- tmp1$dec
Likelihood <- 0
for (k in 1:65) {
  x.t <- 40 * h^process[k, 1:(dec[k]), 1]
  s.t <- 40 * h^process[k, 1:(dec[k]), 2]
  CV <- cv.crra(x.t, s.t, theta, kappa)
  SV <- sv.crra(x.t, s.t, theta, kappa)
  Q <- CV - SV
  T <- length(Q)
  tmp <- ifelse(abb[k] == 1, sum(log((1 - tremble) * pnorm(Q[1:(T -
    1)], mean = 0, sd = sigma) + tremble)) + log((1 -
    tremble) * pnorm(-Q[T], mean = 0, sd = sigma)), sum(log((1 -
    tremble) * pnorm(Q[1:T], mean = 0, sd = sigma) +
    tremble)))
  Likelihood <- tmp + Likelihood
}
Log.like <- Likelihood + prior(Theta, prior)
cat(" =>", Log.like, "\n")
return(Log.like * penFac)
}

```

Note that it is straightforward to change the code for the log-likelihood of the model to go from a model for each subject to a pooled model.¹

¹For a larger subject pool or larger number of iterations, e.g. for Monte-Carlo exercises, the computationally heavy parts of the code could be outsourced using the C++ interface provided by the Rcpp package (Eddelbuettel & Francois, 2011; Eddelbuettel, 2013) and the header files to use the linear algebra environment Armadillo from the R package RcppArmadillo (Eddelbuettel & Sanderson, 2014).

B.3.2. Pre-estimation: Finding posterior modes

First, given the data from the laboratory experiment, we set up an item that contains the different random walks X_t together with the running maximum S_t and the number of times the value of X_t occurred before.

```
process <- array(, c(65, 504, 4))
for (i in 1:65) {
  for (j in 1:504) {
    tmp <- series[i, 1:j]
    process[i, j, ] <- c((log(series[i, j]) - log(40))/log(1.06),
      max((log(series[i, 1:j]) - log(40))/log(1.06)), ifelse(is.na(tmp[j]),
        NA, length(tmp[tmp == tmp[j]])), NA)
    tmp2 <- process[i, 1:j, 1:2]
    process[i, j, 4] <- sum(sapply(1:ifelse(j == 1, 1, dim(tmp2)[1]),
      function(x) {
        if (j == 1) {
          all(tmp2[x] == tmp2[j])
        } else {
          all(tmp2[x, ] == tmp2[j, ])
        }
      })))
  }
}
colnames(process) <- c("x", "s", "multiplicity x", "multiplicity xs")
```

Next we define some global variables that we set for the experiment and that we require for subsequent computations

```
p <- 0.52
h <- 1.06
hazard <- 0.007
delta <- 1 - hazard
K <- 40
x0 <- 40
type <- "crra"
```

Using these primitive parameters, we calculate some secondary parameters, e.g. the term α in the probability to reach a certain level of the process

```
p1 <- 1/(2 * delta * p)
q1 <- (1 - p)/p
zz <- p1 + sqrt(p1^2 - q1)
```

```

alpha <- log(zz)/log(h)
z <- (zz)^(-1)
if (alpha < 1) {
  stop("\n\n\t -- alpha is smaller than 1, but it must be larger! -- \n")
}

```

To find a starting value for the Metropolis-Hastings algorithm, we find the posterior mode of the individual posteriors using the `optim` command (mind the sign of the likelihoods above).²

```

library(parallel)
cores <- detectCores()
clust <- makeCluster(cores)
MLE <- function(i, sp, prior) {
  hyperpars <- prior
  parFac <- pmax(0.01, abs(sp))
  mle <- optim(sp/parFac, fn = LLF, method = "BFGS", data = data[data$individuals ==
    i, ], prior = hyperpars, control = list(fnscale = -1))
  if (1) {
    cat("Restart at first optimum -- try to refine solution\n")
    sp <- mle$par * parFac
    parFac <- pmax(0.01, abs(sp))
    mle <- optim(sp/parFac, fn = LLF, method = "Nelder-Mead",
      data = data[data$individuals == i, ], prior = hyperpars,
      control = list(fnscale = -1))
  }
  mle$par <- mle$par * parFac
  parFac <- 1
  return(mle)
}
clusterExport(clust, c(ls()))
ptm <- proc.time()
res.EU <- clusterApplyLB(cl = clust, x = 1:44, fun = MLE, sp = c(1,
  5, 0.65), prior = c(0.7, 0.3))
laufzeit <- proc.time() - ptm
ptm <- proc.time()
res.Regret <- clusterApplyLB(cl = clust, x = 1:44, fun = MLE,
  sp = c(0.7, 0.1, 5, 0.65), prior = c(0.7, 0.3))
laufzeit2 <- proc.time() - ptm

```

²We parallelized computations across several CPU cores using the package `parallel` in R. This package allows to spawn child processes of R that perform computations simultaneously.

The objects `res.EU` and `res.Regret` are nested lists which contain the returned lists from `optim`. The maxima returned by `optim` are handed over as starting values for the Metropolis-Hastings algorithm in the next step.

B.3.3. Estimation: Posterior simulation via the Metropolis-Hastings algorithm

As was mentioned, we take the posterior modes for each subject and pass it on to the function that performs posterior simulation. Posterior simulation is done independently for each subject.

```
library(MHadaptive)
clust <- makeCluster(cores)
clusterEvalQ(clust, library(MHadaptive))
MH <- function(i, prior) {
  hyperpars <- prior
  parFac <- 1
  par <- posterior.modes[, i]
  mha <- Metro_Hastings(LLF, par, adapt_par = c(100, 100, 0.5,
    0.75), data = data[data$individuals == i, ], prior = hyperpars,
    iterations = 5000, burn_in = 1000)
  return(mha)
}
posterior.modes <- unlist(sapply(1:44, function(x) res.EU[[x]]$par))
# Export EU results to cluster
clusterExport(clust, c(ls()))
# Do MCMC for EU model
metro.EU <- clusterApplyLB(cl = clust, x = 1:44, fun = MH, prior = c(0.7,
  0.3))
posterior.modes <- unlist(sapply(1:44, function(x) res.Regret[[x]]$par))
# Export EU results to cluster
clusterExport(clust, c(setdiff(ls(), "metro.EU")))
# Do MCMC for EU model
metro.Regret <- clusterApplyLB(cl = clust, x = 1:44, fun = MH,
  prior = c(1, 1, 0.5, 100), prior.class = "GH", pref.class = "R")
# Inherit the workspace to each node
```

C Appendix to Chapter 3

C.1. Proofs

Proof of Lemma 5. The expected utility from stopping after $Q_\tau = q$ upticks is given as

$$\Omega(Q_\tau = q) = -u(K) + \mathbb{E} \left[\sum_{t=\tau}^{\infty} \mathbf{1}_{\{s < T\}} u(x_0 h^q h^{Q_t}) \right] .$$

In order to find a closed-form for the latter part of Ω , let

$$W(Q_\tau = q) = \mathbb{E} \left[\sum_{t=\tau}^{\infty} \mathbf{1}_{\{s < T\}} u(x_0 h^q h^{Q_t}) \right] .$$

Then

$$\begin{aligned} W(Q_\tau = q) &= \mathbb{E} \left[\sum_{t=\tau}^{\infty} \mathbf{1}_{\{s < T\}} u(x_0 h^q h^{Q_t}) \right] \\ &= \mathbb{E} \left[\sum_{t=\tau}^{\infty} \mathbf{1}_{\{s < T\}} (x_0 h^q h^{Q_t})^\theta \right] \\ &= h^\theta \mathbb{E} \left[\sum_{t=\tau}^{\infty} \mathbf{1}_{\{s < T\}} (x_0 h^{q-1} h^{Q_t})^\theta \right] \\ &= h^\theta W(q-1) . \end{aligned}$$

Consequently, W is a first-order difference equation and its solution is of the form

$$W(q) = \beta h^{\theta q} , \tag{C.1}$$

where β is a parameter that has to be determined by an additional condition. To find the value for β , note that the expected utility from stopping at $Q_\tau = q$ is the

sum of the current utility from stopping $u(x_0 h^q)$ plus the expected sum of future utilities, i.e.

$$W(q) = u(x_0 h^q) + \delta [p(q) W(q+1) + (1-p(q)) W(q-1)] .$$

Substituting (C.1), yields

$$\begin{aligned} \beta h^{\theta q} &= (x_0 h^q)^\theta + \delta [p(q) \beta h^{\theta(q+1)} + (1-p(q)) \beta h^{\theta(q-1)}] \\ \beta &= \frac{x_0^\theta}{1 - \delta [p(q) h^\theta + (1-p(q)) h^{-\theta}]} . \end{aligned}$$

Collecting the results, we find that

$$\Omega(q) = -K^\theta + \frac{h^{\theta q} x_0^\theta}{1 - \delta [p(q) h^\theta + (1-p(q)) h^{-\theta}]} .$$

□

C.2. Figures

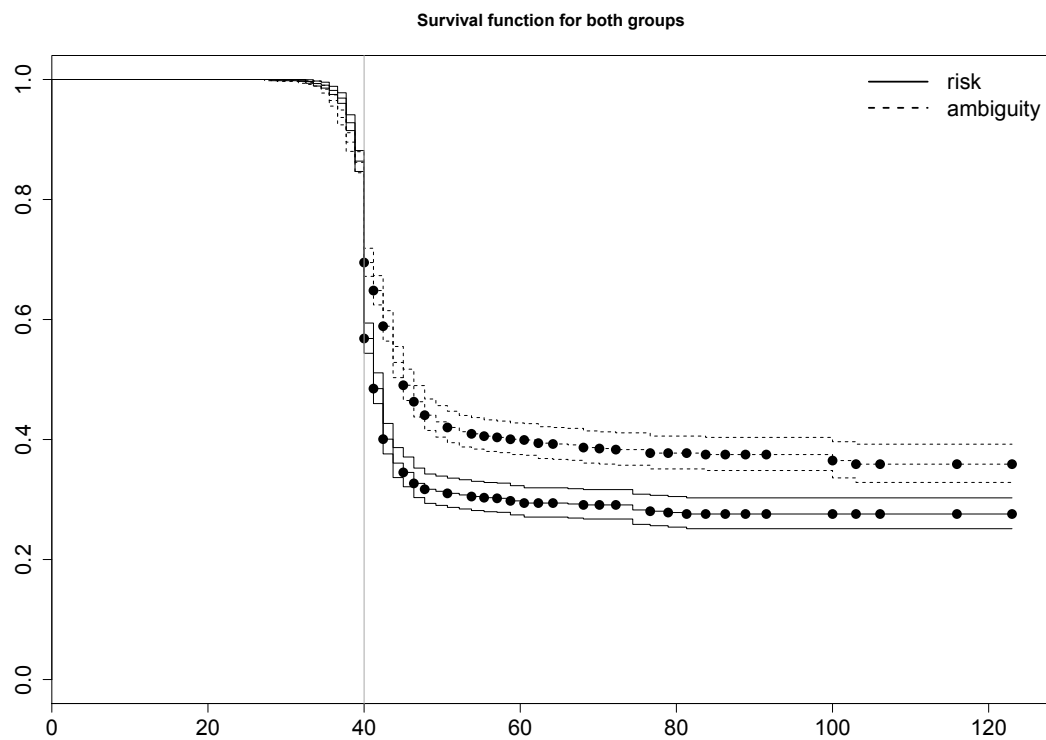


Figure C.1.: Estimate of the survival function by group. The dashed vertical line indicates the initial value of the payoff process.

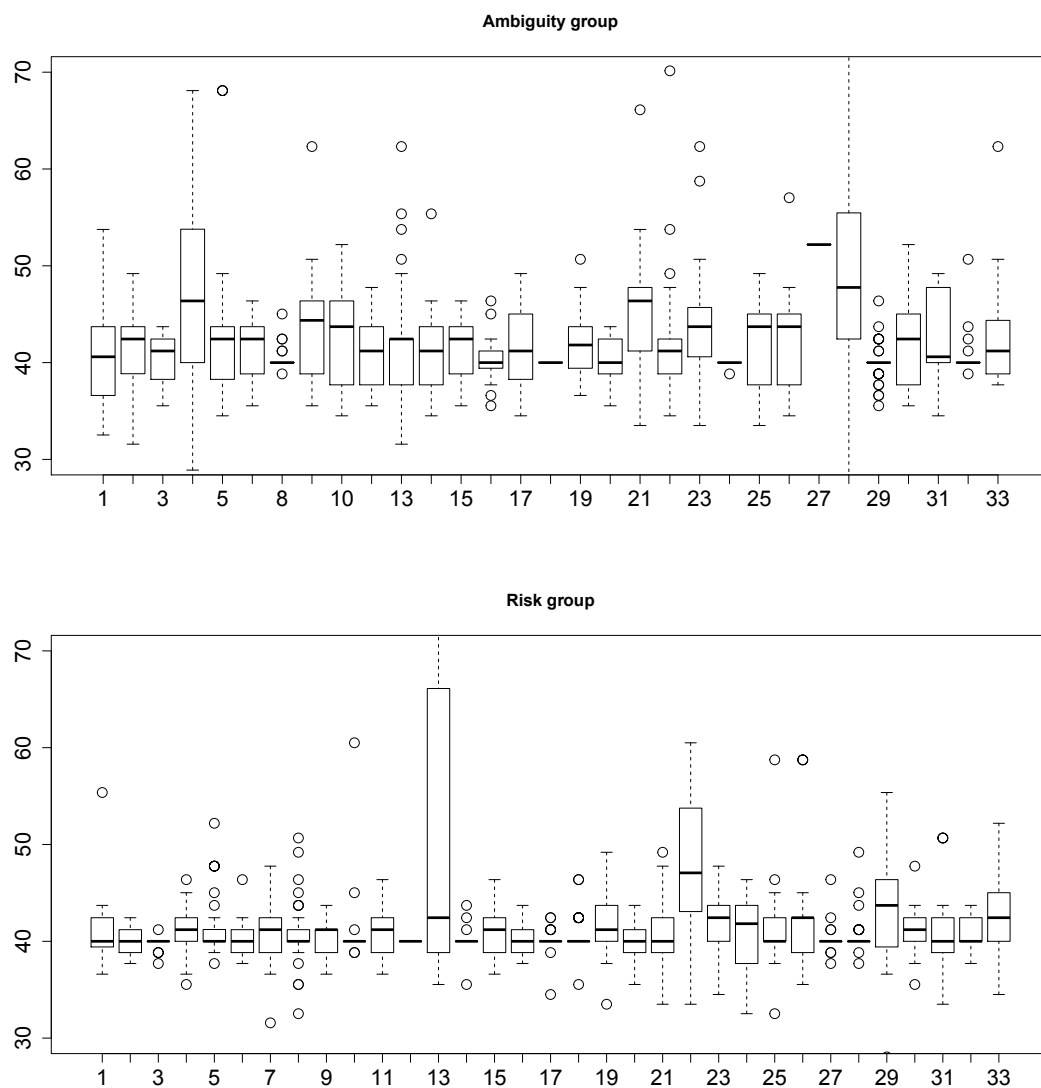


Figure C.2.: Boxplots of observed reservation profits over 45 rounds by subject.

Bitte wählen Sie auf welche Farbe die Produktion in der nächsten Runde eingestellt werden soll:

☐ Auf Rot

☐ Auf Schwarz

Figure C.3.: Screen for color choice (in German).

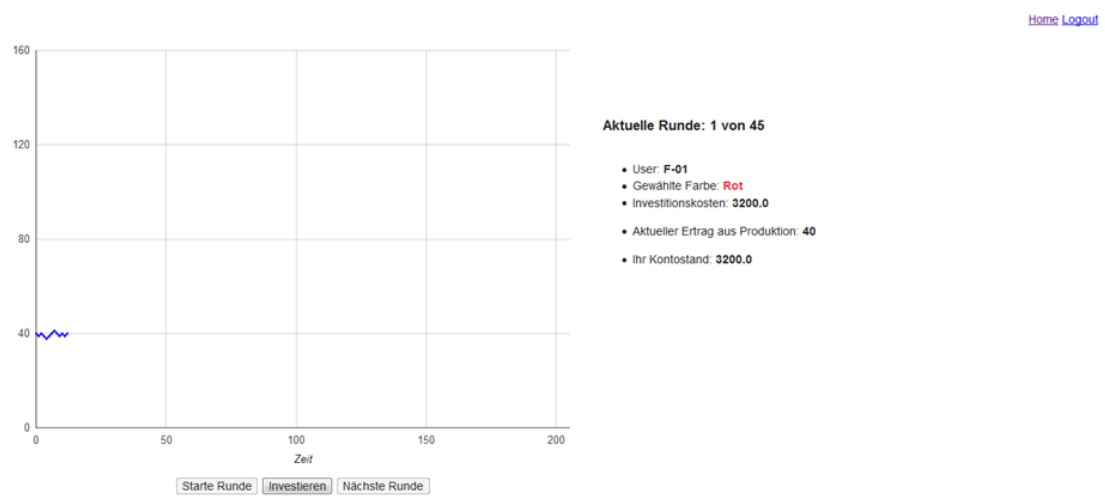


Figure C.4.: Investment screen (in German).

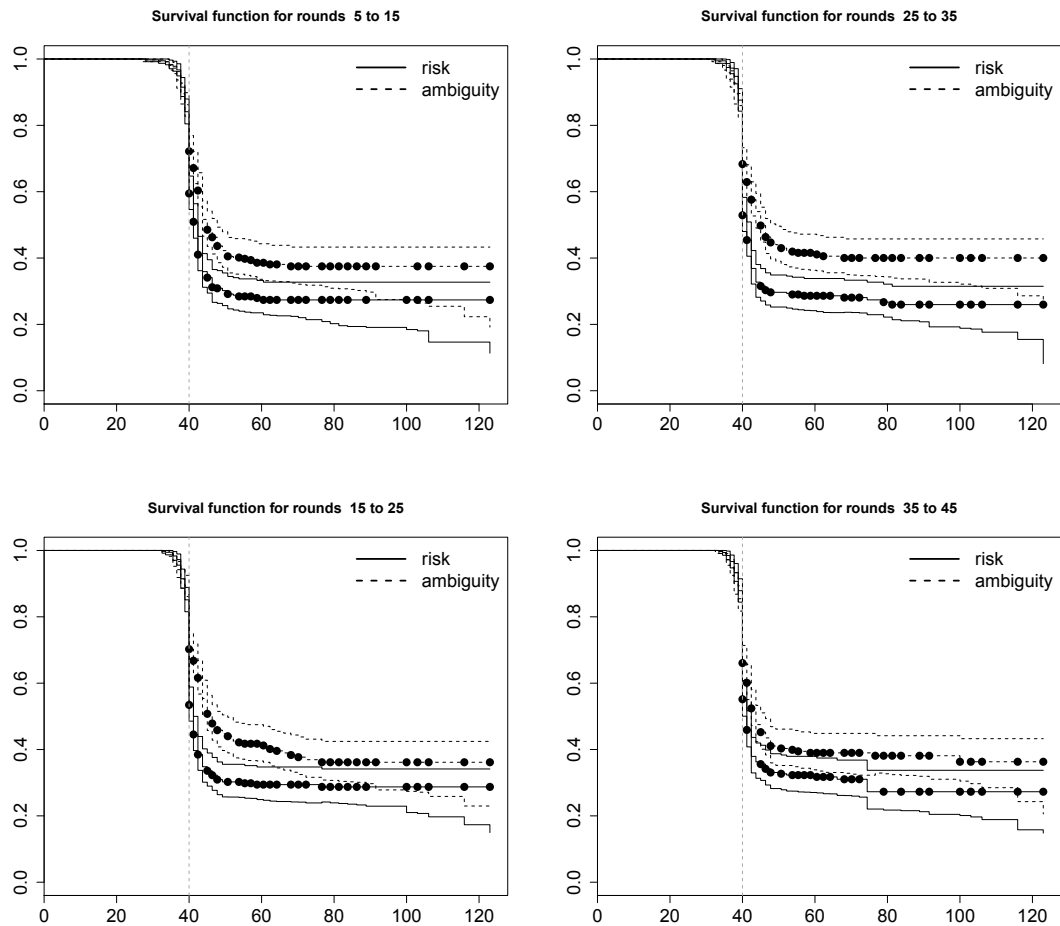


Figure C.5.: Estimate of the survival function by group and over blocks of rounds. Standard errors are not clustered by subject but lower limit is modified based on an "effective n " argument. The extra width is obtained by multiplying the usual variance by a factor m/n , where n is the number currently at risk and m is the number at risk at the last death time (see reference manual of package 'survival' for R). The dashed vertical line indicates the initial value of the payoff process.

C.2.1. Tables

Table C.1.: Results from mixed proportional hazard model for rounds 5 to 15.

	coeff.	exp(coeff.)	SE	z-stat.	Pr(> z)
Ambiguity effect	-0.42	0.66	0.19	-2.15	0.03
LR test for random effects			-76.76	p: 0.00	

Notes: Sample size N=728. Efron approximation for ties.

Table C.2.: Results from mixed proportional hazard model for rounds 15 to 25.

	coeff.	exp(coeff.)	SE	z-stat.	Pr(> z)
Ambiguity effect	-0.46	0.63	0.22	-2.10	0.04
LR test for random effects			-105.14	p: 0.00	

Notes: Sample size N=726. Efron approximation for ties.

Table C.3.: Results from mixed proportional hazard model for rounds 25 to 35.

	coeff.	exp(coeff.)	SE	z-stat.	Pr(> z)
Ambiguity effect	-0.51	0.60	0.24	-2.15	0.03
LR test for random effects			-131.18	p: 0.00	

Notes: Sample size N=726. Efron approximation for ties.

Table C.4.: Results from mixed proportional hazard model for rounds 35 to 45.

	coeff.	exp(coeff.)	SE	z-stat.	Pr(> z)
Ambiguity effect	-0.33	0.72	0.24	-1.35	0.18
LR test for random effects			-116.04	p: 0.00	

Notes: Sample size N=660. Efron approximation for ties.

C.3. Instructions

Welcome!

Please read these instructions carefully.

Please remain seated during the whole experiment. Do not communicate with any other participant and remain calm.

Should you have questions regarding the experiment or the instructions, please raise your hand and one of the Experimentators will come to your place.

After you have finished the experiment, please also remain seated. Also please do not log out of the computer-based experiment before we have paid you.

The experiment today consists of 45 rounds in which you will have to make one decision each. After you have finished all 45 rounds, we will pay you a 7 Euro show-up fee plus the amount of points that you have earned in ONE randomly determined round.

Points are converted to Euros according to the simple formula:

You complete payoff in EUR = $0.002 \times (\text{No. of points earned in ECU}) + 7.00 \text{ EUR Show-up fee}$.

Which of the 45 rounds will be used to determine your payoff is completely random. The computer will draw with equal probability a number between 1 and 45 to determine your payoff. The result will be displayed on a final screen.

The basic idea

In this experiment you will decide when (if ever) to invest into a factory. In case you decide to invest into the factory in a given round, the factory produces a fictitious good. You earn proceeds from selling this product over time. In each round you are endowed with 3,200 ECU. Investment into the factory involves fixed cost (for building the factory) of 3,200 ECU. These costs only have to be incurred once.

In each of the 45 rounds you will play, you may only invest into the factory once, not multiple times.

You may earn more than your initial endowment from your investment into the factory. You may, however, also earn LESS than your initial endowment.

Your computer screen will display useful information to support your investment decision in the following diagram:

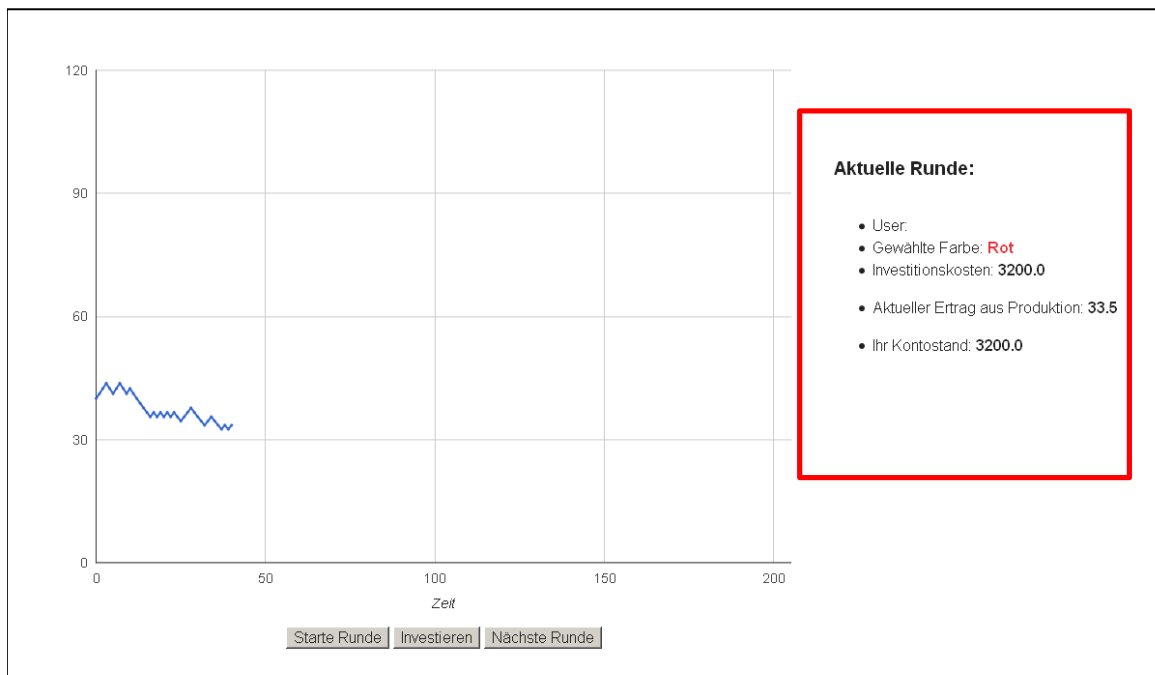


Figure 1

As you can see from the picture, there are three buttons available to you. To begin a given round, click on the button "Begin round". As soon as you have clicked this button, the computer will start to display the evolution of potential profits from selling the product. Every second there are two ticks. The starting value is always 40. From there on, profits rise or fall from their previous value by 3%. Assume that the current value is equal to 100. The next value will then be either $100 \cdot 1.03 = 103$ or it will be $100 \cdot (1/1.03) \approx 97.09$.

As you can also see in the picture above, the computer displays the evolution of profits as a jagged blue line. On the horizontal axis time in seconds is depicted. The vertical axis depicts the last value of the profit process in ECU. The current value of the profit process is the right-most tip of the blue line (in the picture above it is equal to 33.5 ECU). There is additional information about the current round, summarized in the red box at the right-hand side of the screen.

As soon as you have started a round, you may seize the investment by clicking the button „Invest“. Pressing „Invest“ has two immediate consequences: (i) you have to incur the investment cost of 3,200 ECU and your account balance drops to zero and (ii) you will earn the sum of all current and future profits until the end of the round. For example, suppose you invest in the situation depicted in Fig. 1. In that case, you would immediately earn the current value 33.5 ECU plus all future values until the end of a round. To highlight this, the jagged line will turn green from the point where you invested.

In case, you decide not to invest in a given round, your final payoff will be zero ECU. In case you invested in a given round, your final payoff equals the sum of profits earned after investment.

How long a round lasts is RANDOM. Consequently, the payoff you are able to earn from your investment is random. The computer will determine prior to each period, i.e. twice every second, whether a round ends or not. There is a constant and fixed probability of 0.7% that a round ends after a given period. You notice a round has ended once the computer stopped extending the jagged line and no new value is depicted.

With the end of a round you will also lose your option to investment into the factory, unless you have done so before. If you invested before, the end of the round means also that you stop earning profits from your investment.

Therefore on the one hand you should not hesitate too long to invest, should you be convinced that investment is profitable. On the other hand you should not invest prematurely, in case you feel you cannot gauge the risk of the investment.

As soon as a given round has ended, the button “Next round” is activated and you may proceed to the next round.

Details

How much you will earn from selling you are the product produces by the factory, depends on whether or not you happen to match consumers’ preference for the COLOR OF THE PRODUCT. There are two

possible colors for the product: RED or BLACK. Prior to each round, the computer randomly determines which color is the preferred color of consumers. You will not be informed about the outcome. The factory is only able to produce goods of a particular color, not both at the same time.

Before the start of each round, you will have to fix which color machines will be producing. After you have fixed the setup, it cannot be changed during a round. You may only change it between rounds. In case you guess the preferred color correctly, it is more likely that the profits from selling the product will rise over time. Conversely, in the case you did not guess the preferred color correctly, it will be more likely that profits fall over time.

More precisely:

- In case you guess the preferred color correctly, profits will have a **57% chance** to increase by 3% and decrease with **43% chance** by 3% each tick.
- In case you guess the preferred color incorrectly, profits will have a **43% chance** to increase by 3% and decrease with **57% chance** by 3% each tick.

As mentioned before, the computer determines the preferred color for each round separately. The preferred color is RED with probability q and BLACK with probability $1 - q$.

TEXT FOR AMBIGUITY (TREATMENT) GROUP:

You do not have any further information about what the probability q exactly is. All you know is that it equals the average share of rainy days over a year in Jakarta (Capital of the Republic of Indonesia). That is, the more often it rains in Jakarta, the more likely it is that the preferred color will be RED.

For example, suppose the average number of rainy days in Jakarta within a year is 251 days. Then q would equal 251 divided by 365 or roughly 68%. If the average number of rainy days in Jakarta was 116 days, q would equal 116 divided by 365 or roughly 32%. If you are convinced that it is more likely that RED will be the preferred color, you should choose to set your machines to produce red products.

Conversely, if you are more convinced BLACK will be the preferred color, you should choose to set your machines to produce black products.

TEXT FOR RISK (CONTROL) GROUP:

You know that q is equal to 0.5 or 50%.

Summary

To wrap it up again:

- You will have to make investment decision over the course of 45 rounds.
- Investing into the factory may earn you more than the initial building cost, but it also bears the risk of earning you less than the initial building cost.
- In case you guess the favorite color of consumers correctly, you are likely to earn more than otherwise.

Should you have no further questions regarding the experiment, please switch on the computer screen in front of you and log into the experiment. Your login is equal to your seat number, please confirm your entry with the button “Login”. Please note that your login is case sensitive. You also have to include a minus sign between the initial letter and your number.

Your seat number is: A-22.

The experiment will then start with the choice for the color for the first round.

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Ehrenwörtliche Erklärung

Die ersten beiden Kapitel basieren auf einem gemeinsamen Projekt mit Philipp Strack. Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Ort, Datum

Unterschrift